بسم الله الرحمن الرحيم

Nile University
جـامعة النيـل

Qubits of Light: *Singles & Entangled*

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Photons realizes communication qubits

Photons :

- Are chargeless
- Do not interact very strongly with each other, or even with most matter.
- Are guided along long distances with low loss in optical fibers, atmosphere, or free space
- Are delayed and manipulated efficiently in room temperature using traditional optics phase shifters, phase retarders, mirrors, beam splitters…etc.

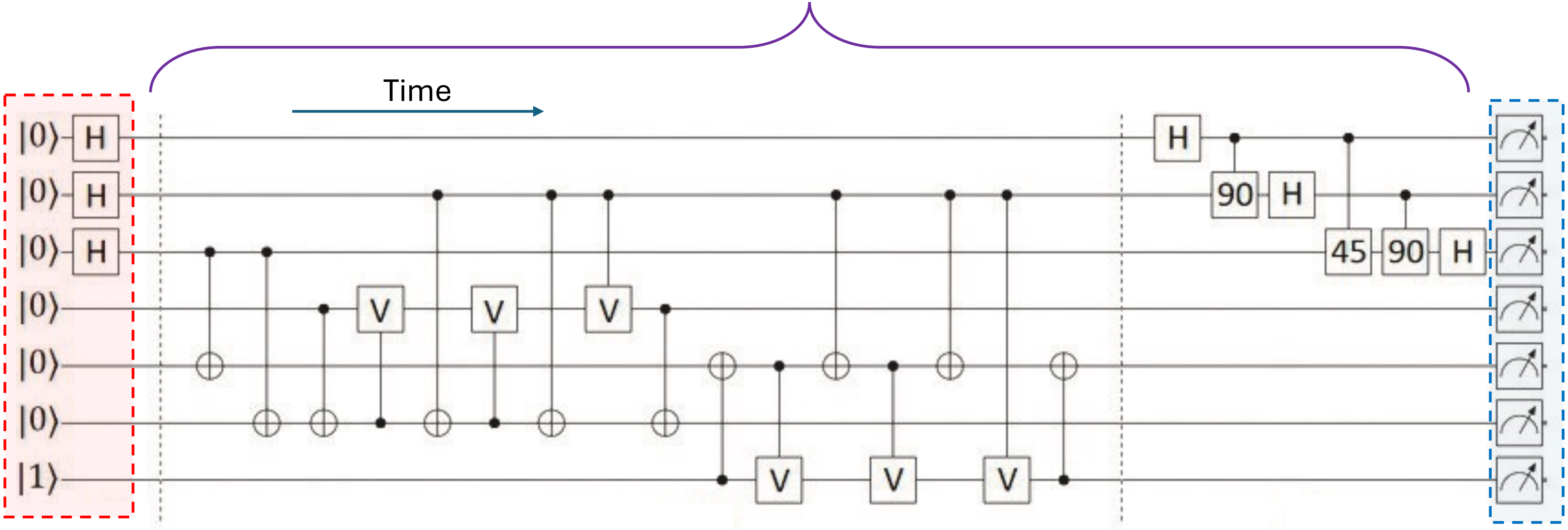
Quantum photonic systems

There is no need for special experimental facilities in Labs **(Readily implementable in Egypt)**

Steps of Quantum Computing

(2) Processing quantum states

- Quantum gates : Unitary rotations



(1) Initializing quantum states

- Single-photon generation (and preparation)

(3) Measuring quantum states

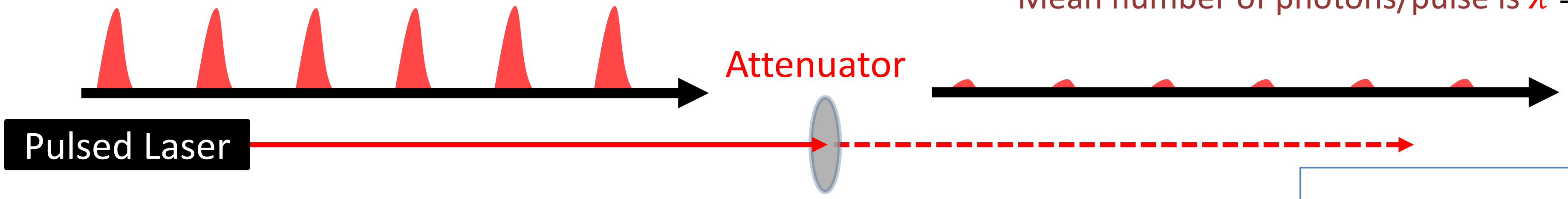
- Projection
- Single-photon detection

How to generate single photons?

1- Generation of Single photons: Faint laser pulses

It's important in photonic quantum computing to have a single-photon source producing Single-photon Fock state: $|n\rangle$ with $n = 1$. (also called photon number state)

Mean number of photons/pulse is $\lambda = E_{pulse}/h\nu$

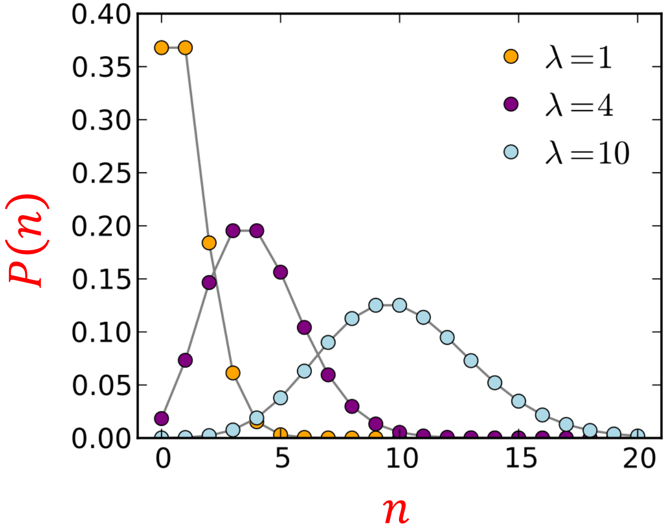


Very good approximation of laser pulse is the Coherent state $\dashrightarrow$
n Photons are thermally distributed within the coherence time

coherent state:

$$|\alpha\rangle = \sum_{n=0}^{\infty} \sqrt{e^{-\lambda}\frac{\lambda^n}{n!}}\,|n\rangle$$

Poisson distribution



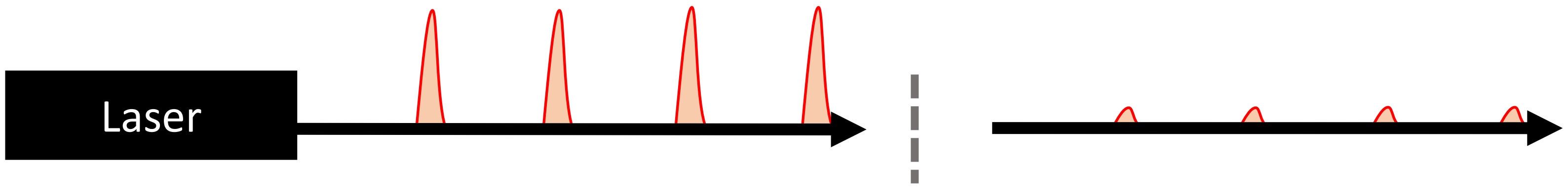
Probability of n photons/pulse given that mean photon number is λ

$$P(n) = e^{-\lambda}\frac{\lambda^n}{n!}$$

Poisson distribution

Faint laser is not ideal Single-photon source

Example: for $\lambda = 0.1$
$P(0) \approx 90\%$ (empty pulses)
$P(1) \approx 9\%$ (single-photon pulse)
$P(>1) \approx 1\%$ (more than a photon / pulse)



Number of photons/pulse is described by: $|\alpha\rangle$

Example:

$\lambda = \mathbf{0.1}$, in this case:

$$|\alpha\rangle = \sqrt{0.9}\,|0\rangle + \sqrt{0.09}|1\rangle + \sqrt{0.002}|2\rangle + \cdots$$

This approximate the single photon Fock state $|1\rangle$, but with:

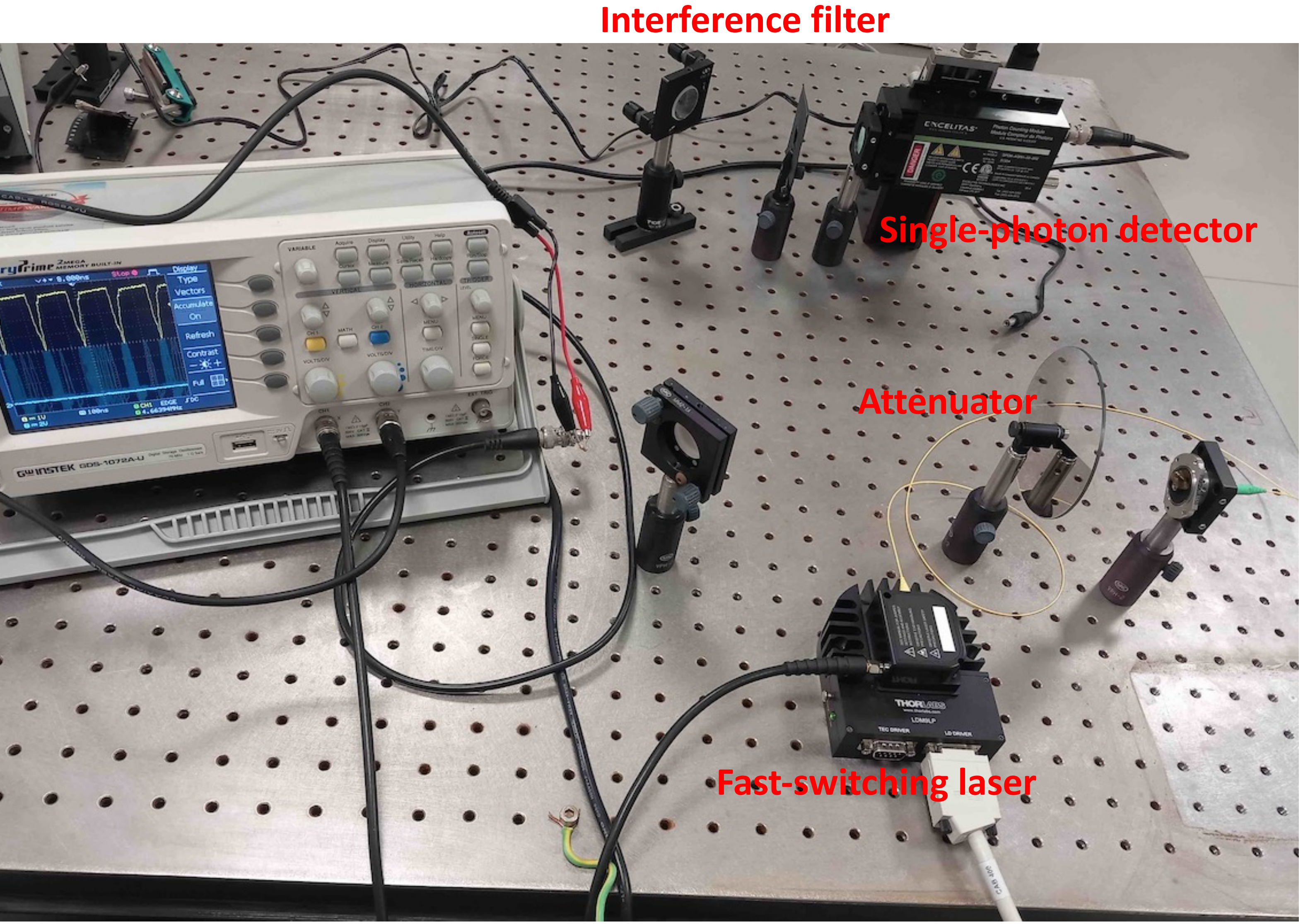
- 90% empty pulses (this is not a big problem!!)
- 9% single-photon pulses
- 1% more than a photon. (A problem in some protocols)

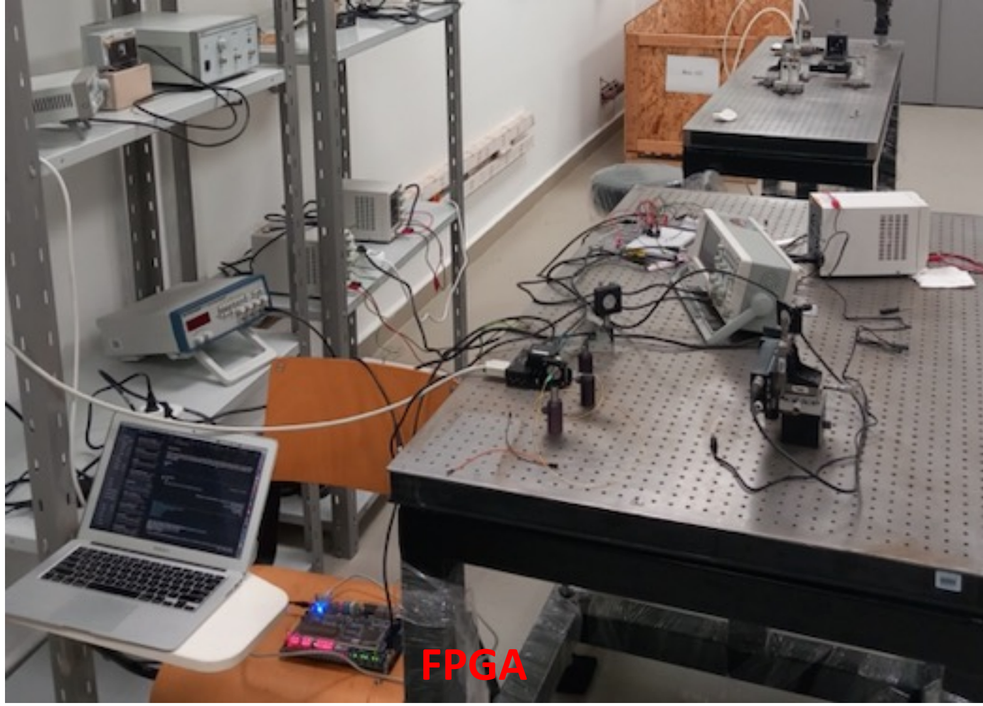
One photon per pulse

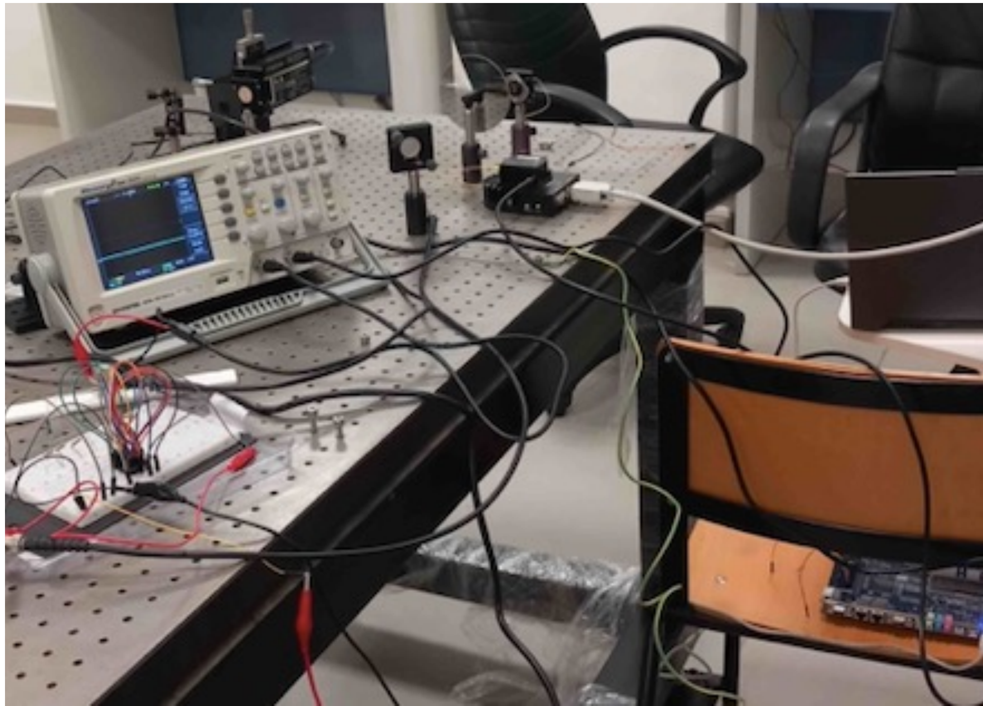
More than one photon per pulse

Ignoring the empty pulses, there is a possibility of about 90% success and 10% failure.

No indication whether the pulse is empty or occupied. Therefore, two of this source cannot be synchronized.







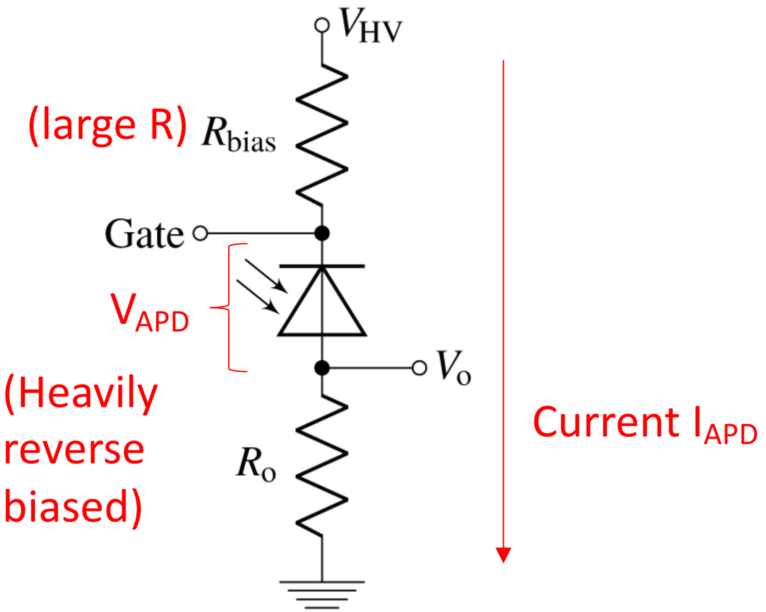
Quantum Communication Lab.

How to detect single photons?

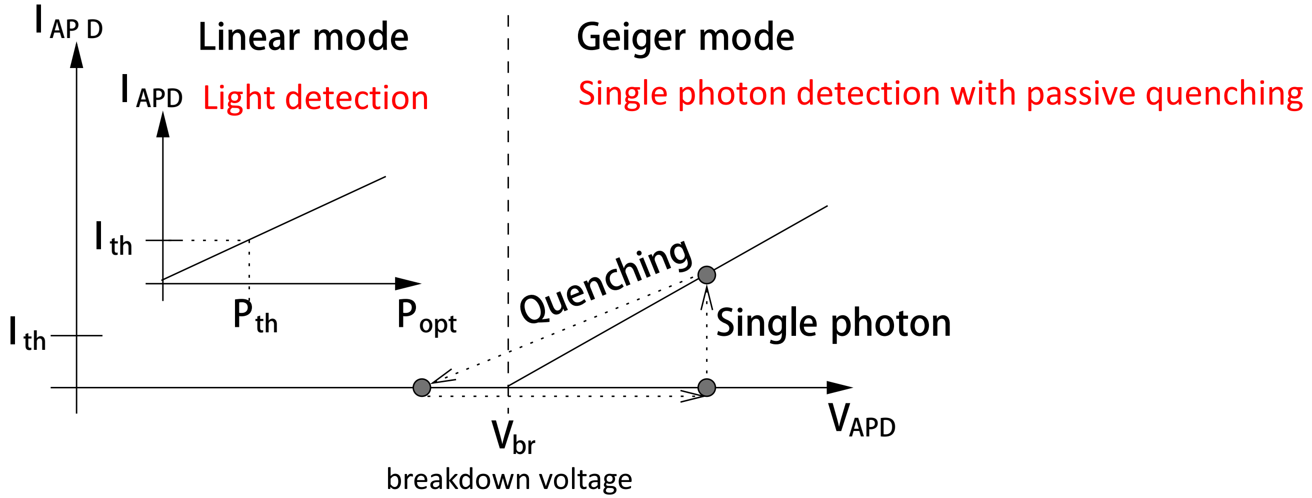
Detection of Single photons

Avalanche photodetectors (APDs) outputs an electronic pulse in response to one input photon (producing an electron-hole pair)

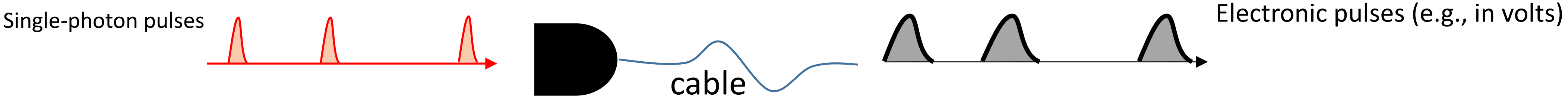
APD Circuit



Operation modes of APD



Single-photon avalanche diode (SPAD) operating in Geiger mode



13.1 The photon (zero rest mass, chargeless)

carries electromagnetic **energy** and **momentum**, as well as **spin angular momentum (SAM)** associated with its **polarization** properties. It can also carry **orbital angular momentum (OAM)**.

In other words, Degrees of freedom (DoFs) of light are

- space/momentum (linear momentum and orbital angular momentum),
- Frequency (energy)/time,
- Polarization (SAM)

Electromagnetic field can be fully represented as a superposition of discrete orthogonal modes in every optical DoF

The electric-field vector, $\varepsilon(r,t) = \mathrm{Re}\{E(r,t)\}$, can therefore be expressed in terms of the complex electric field

$$E(r,t) = \sum_q A_q \; U_q(r) \; \exp(i2\pi\nu_q t) \; \hat{\mathbf{e}}_q$$

The q^{th} mode has:

Complex amplitude A_q
Related to field amplitude, initial phase

frequency ν_q

polarization along a unit vector $\hat{\mathbf{e}}_q$

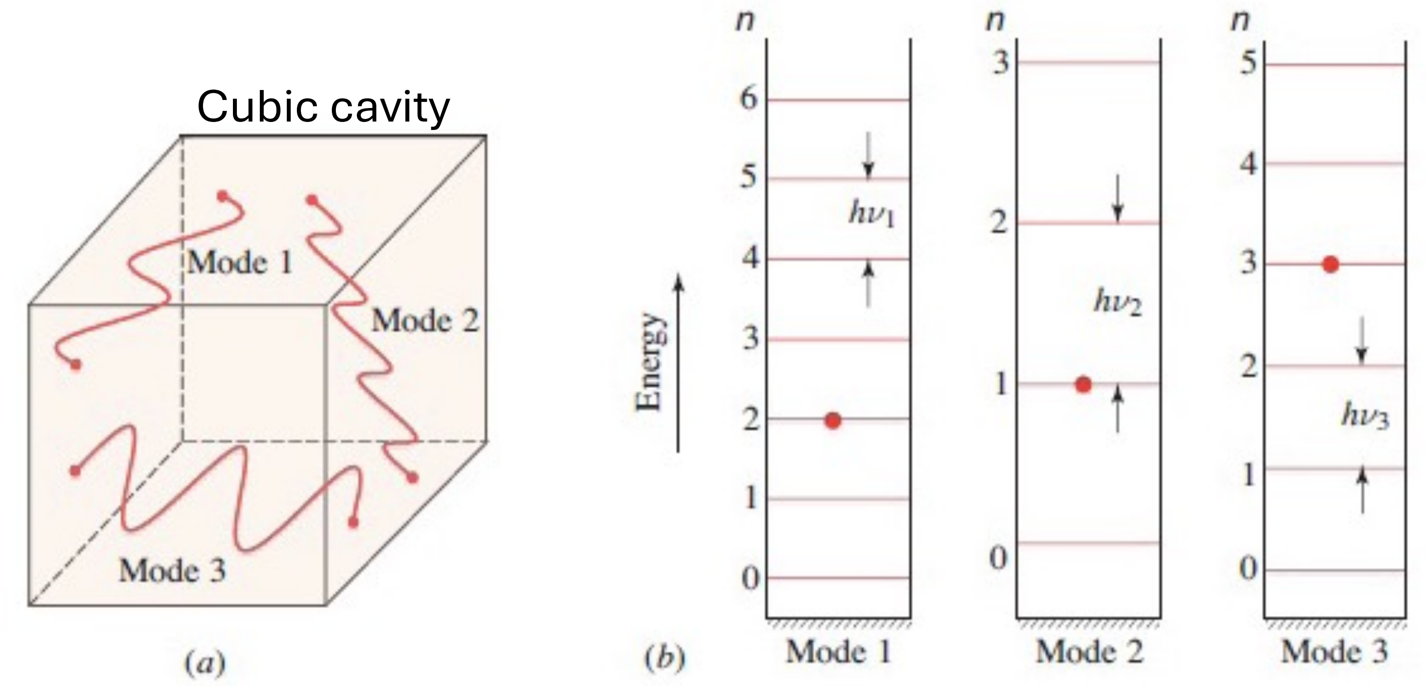
Spatial distribution characterized by complex function $U_q(r)$,
normalized such that $\int_V |U_q(r)|^2 \, dr = 1$.
This distribution can be gaussian, LG,... modes
$U_q(r)$ includes also terms like $\exp(ikz)$ that describes the propagation direction

Note : You can think about it like the description of any arbitrary periodic function as Fourier series with weighted superposition of orthogonal functions

Illustrated:

- Freq.,
- Polarization,
- Direction

of EM mode



Illustrated:

No. of photons in every EM mode,

Or equivalently the energy per mode

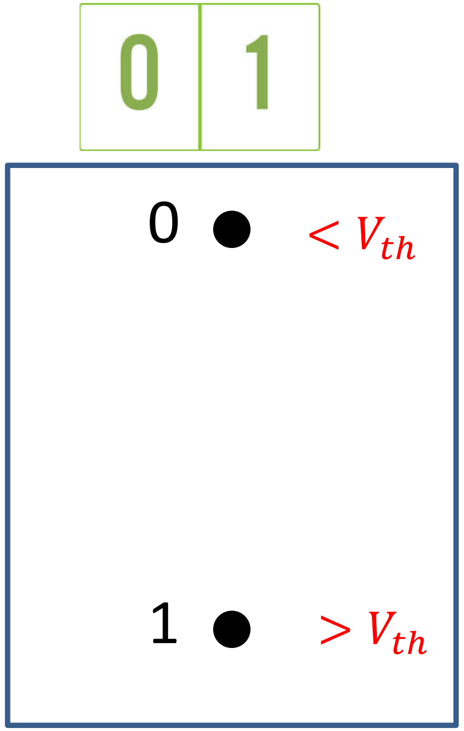
← Vacuum (no-photon) state has energy = $\frac{1}{2}h\nu$

Figure 13.1-1 (*a*) Schematic of three electromagnetic modes of different frequencies and directions in a cubic resonator. (*b*) Allowed energy levels of three modes in the context of photon optics. Modes 1, 2, and 3 have frequencies ν_1, ν_2, and ν_3, respectively. In the example presented in the figure, modes 1, 2, and 3 contain $n = 2$, 1, and 3 photons, respectively, as represented by the filled circles.

Note: In Quantum Optics, the space of bosonic modes is spanned by number (Fock) states: $|0\rangle, |1\rangle, |2\rangle, |3\rangle \ldots, |n\rangle, n$ *is photons number*

Bits & Qubits

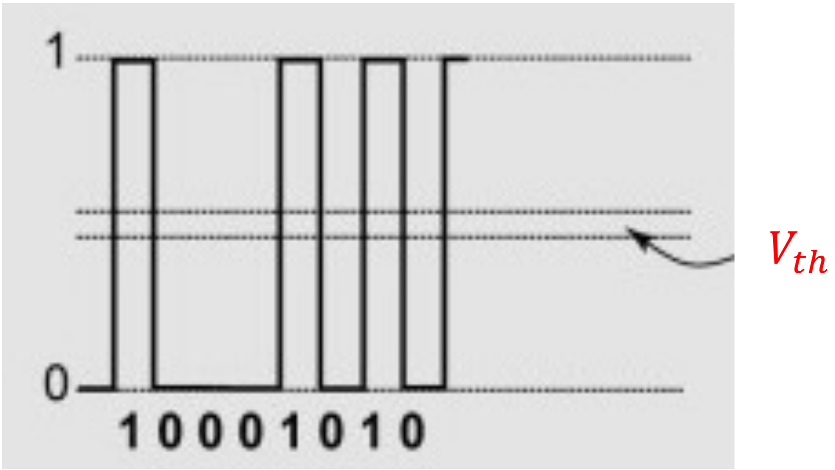
Bits



Discrete two-state classical space

Examples: 1- Amplitude encoding

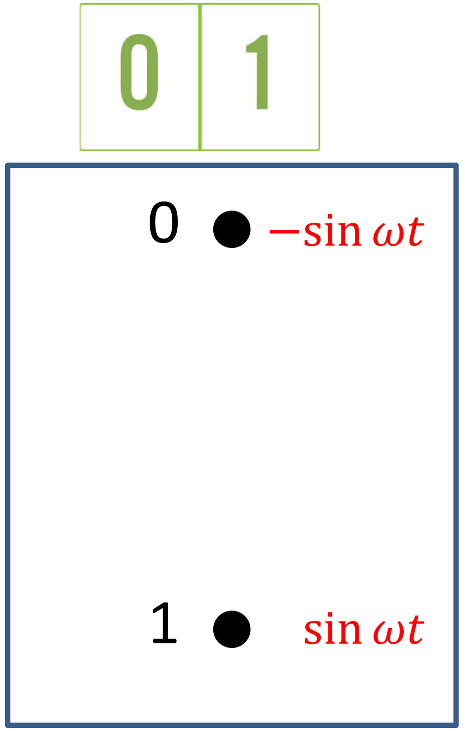
Amplitude shift keying



0, 1 are Fully distinguishable

Bits & Qubits

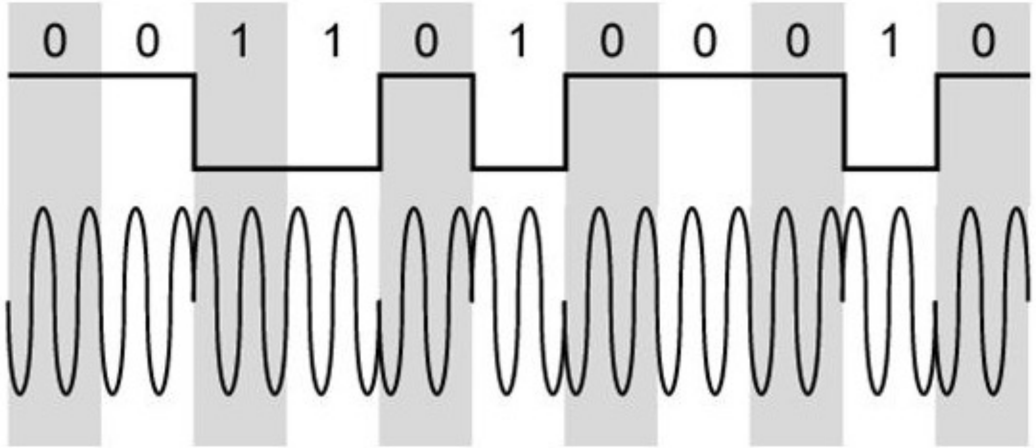
Bits



Discrete two-state classical space

Examples: 2- Phase encoding

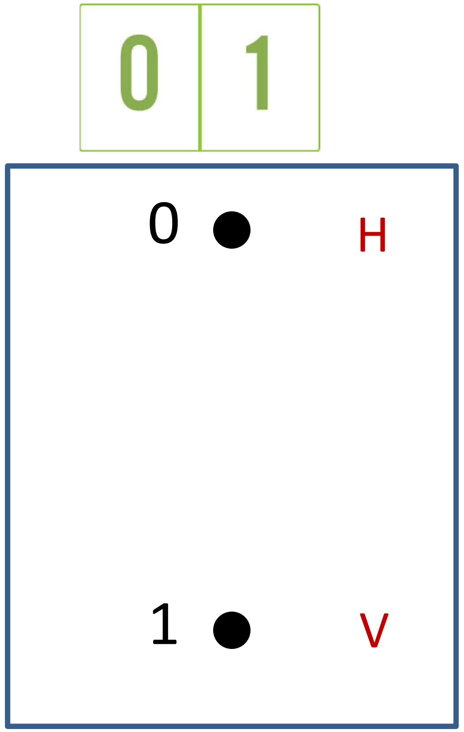
Binary phase shift keying



0, 1 are Fully distinguishable

Bits & Qubits

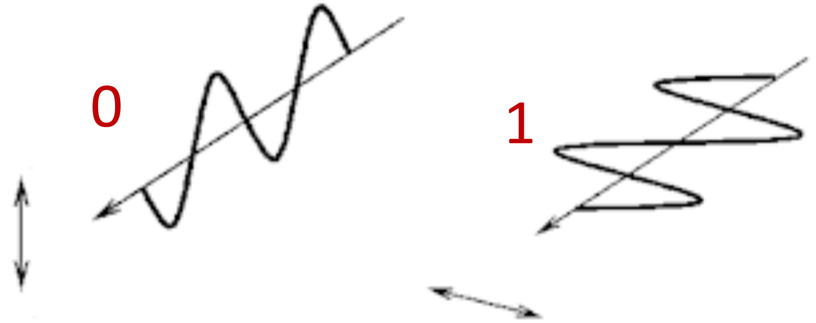
Bits



Discrete two-state classical space

Examples: 3- Polarization encoding

Polarization shift keying



V : Vertical polarization

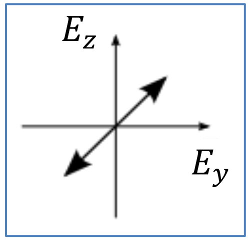
H : Horizontal polarization

0, 1 are Fully distinguishable

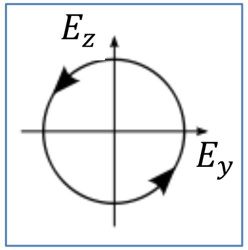
Photonic Qubits can be realized in time

Quantum bit (Qubit)

Naturally, Quantum systems are in "coherent superposition" of possible modes



Linear



Circular

Example: Polarization of a single photon

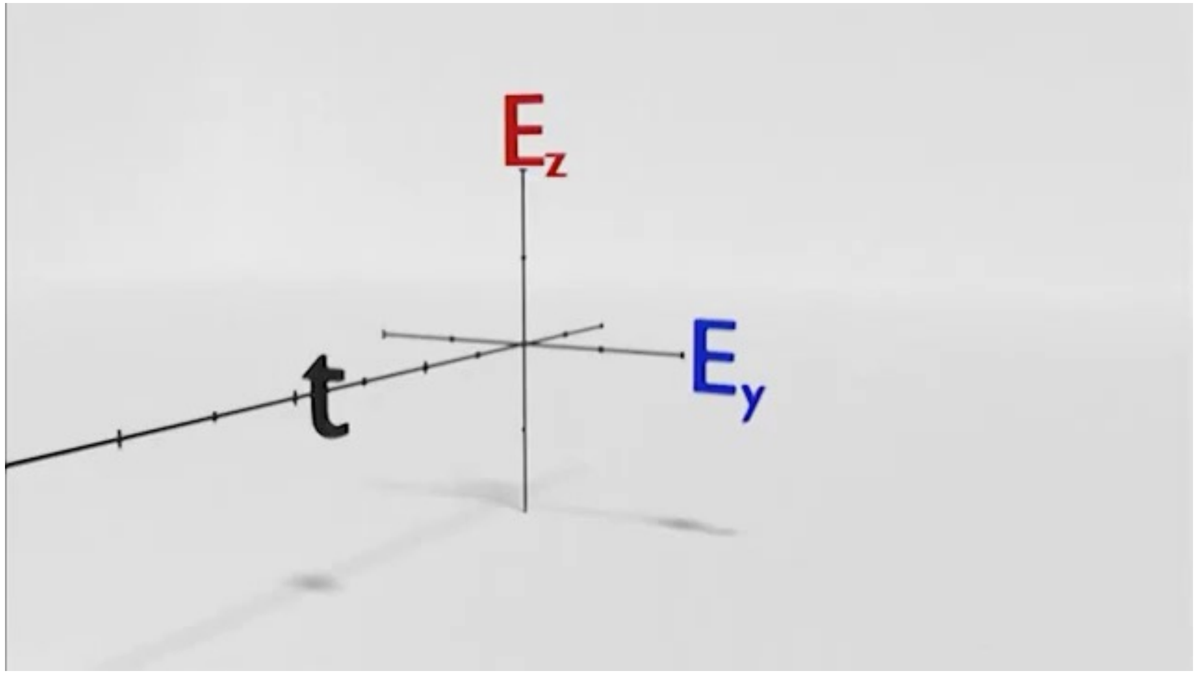
Polarization of a single photon evolves as (*complex*) coherent superposition of two basis states, here E_y, E_z **(polarization is a 2-D property)**

Coherent superposition:
* Photonic modes are added,
** while keeping their relative phase

Orthogonal Polarization modes

$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$$

Complex amplitudes



polarization wavefunction is normalized : $\sqrt{|\alpha|^2 + |\beta|^2} = 1$

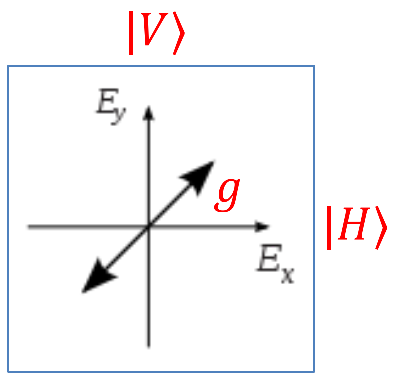
Photonic Qubits

Polarization as a qubit

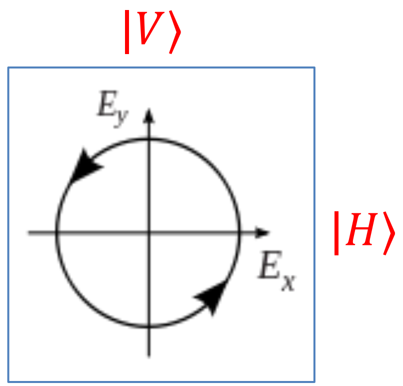
$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi\rangle = \alpha\begin{bmatrix}1\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\alpha\\\beta\end{bmatrix}$$



Linear



Circular

Linear-polarization state: $|linear\rangle = \cos g\,|H\rangle + \sin g\,|V\rangle$

Examples of polarization states:

Diagonal $|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$

Anti-Diagonal $|A\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$

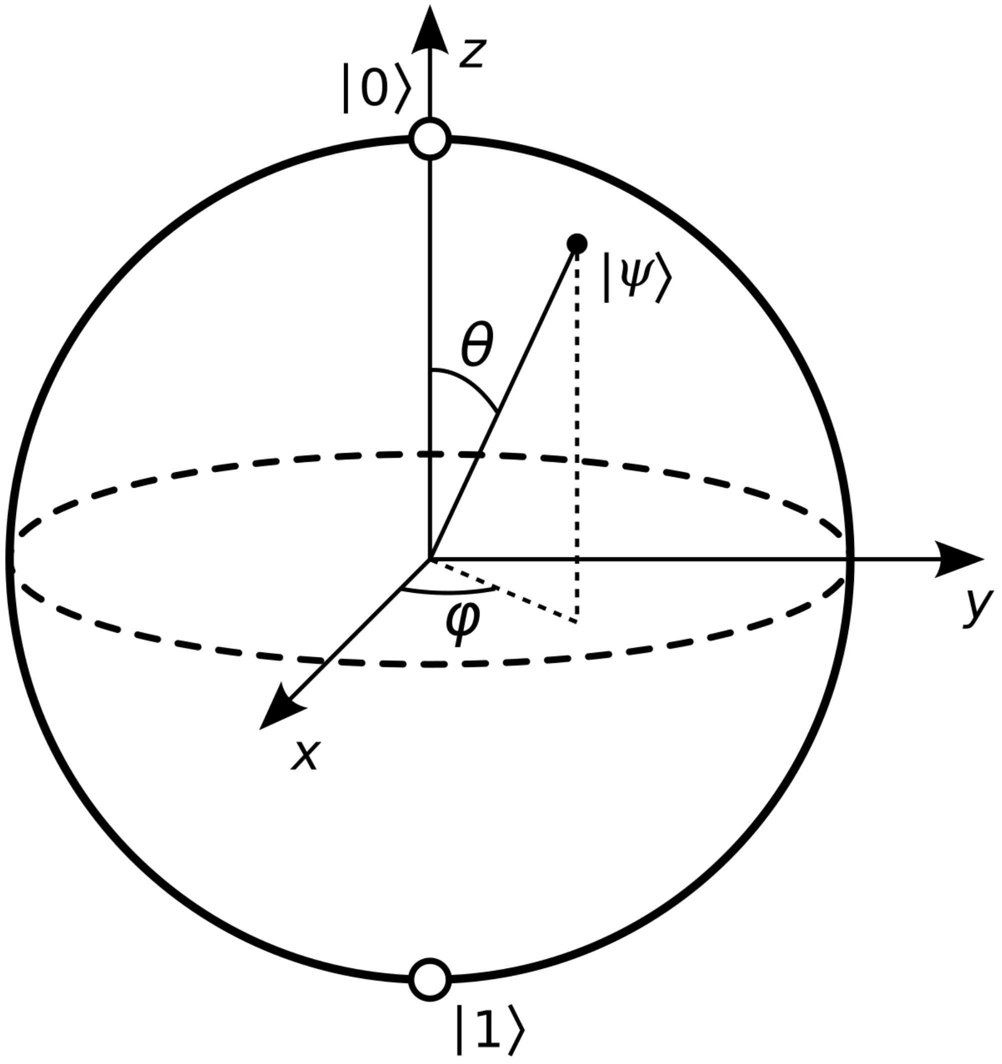
Right-circular $|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle)$

Left-circular $|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$

Qubit $|\psi\rangle$: has a continuum of possible values in C^2, the complex space of dimension 2

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

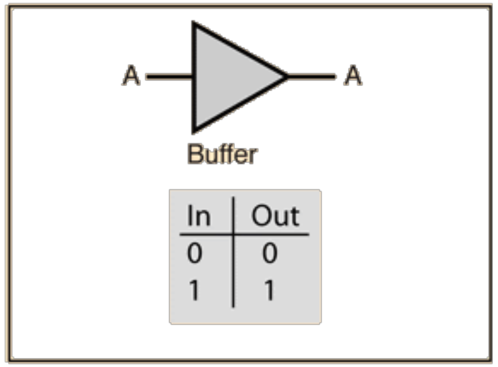
θ: polar angle $\in [0, \pi]$
ϕ: azimuthal angle $\in [0,2\pi]$

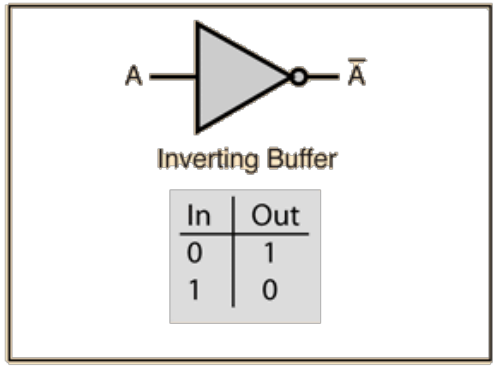


Bloch sphere

Bits & Qubits : Gates

1-bit gates





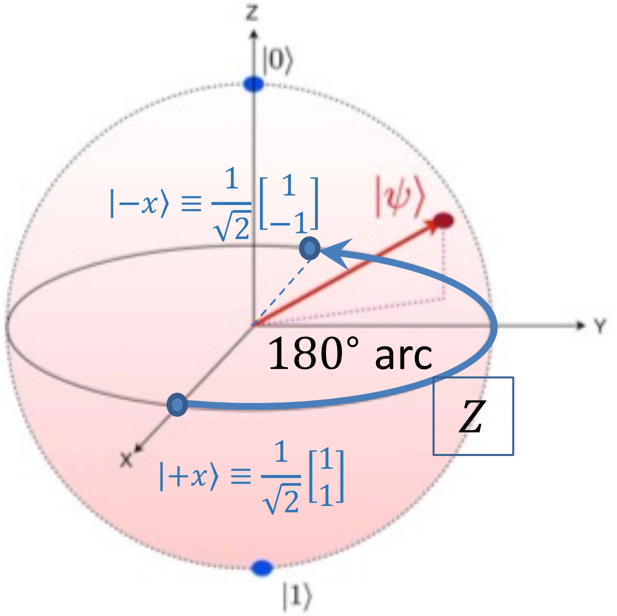
1-Qubit gate :

Infinite, any unitary operator $U(2)$ is a gate!

Bits & Qubits : Gates

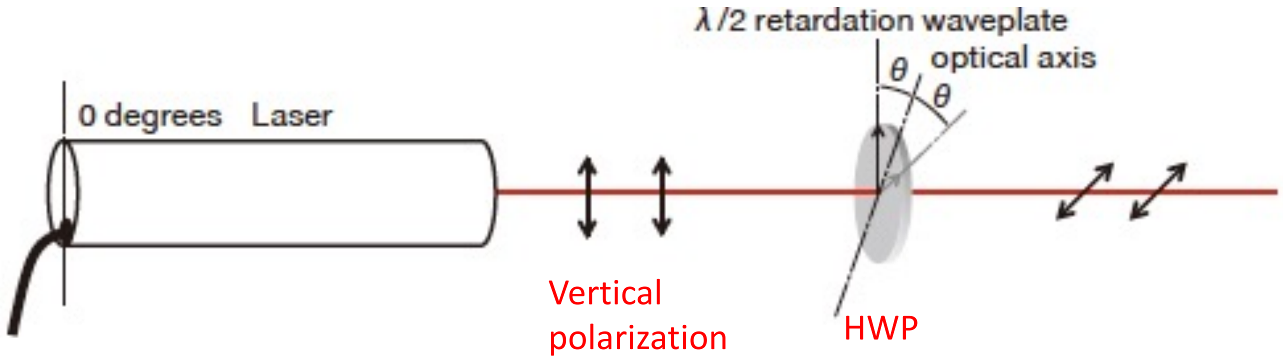
1-Qubit gate : Important examples:

	Circuit symbol	Matrix representation	
Hadamard	H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	
Pauli-X	X	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	**(Quantum NOT gate)** **180° rotation about x axis**
Pauli-Y	Y	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	**180° rotation about y axis**
Pauli-Z	Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	**180° rotation about z axis**
Phase	S	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	**90° rotation about z axis**
$\pi/8$	T	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	**45° rotation about z axis**

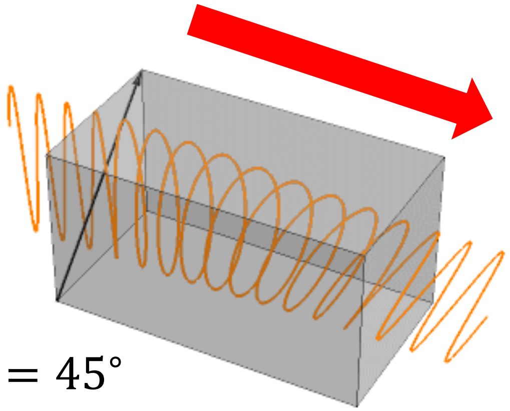


$$Z|+x\rangle \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Half-wave plate (HWP) acting on polarization state



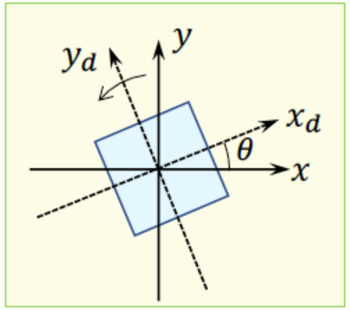
HWP performs unitary rotation of polarization state

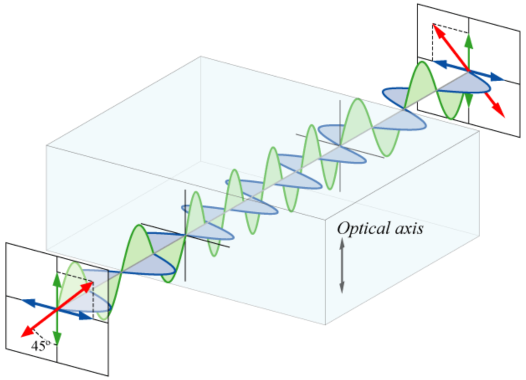


$\theta = 45°$

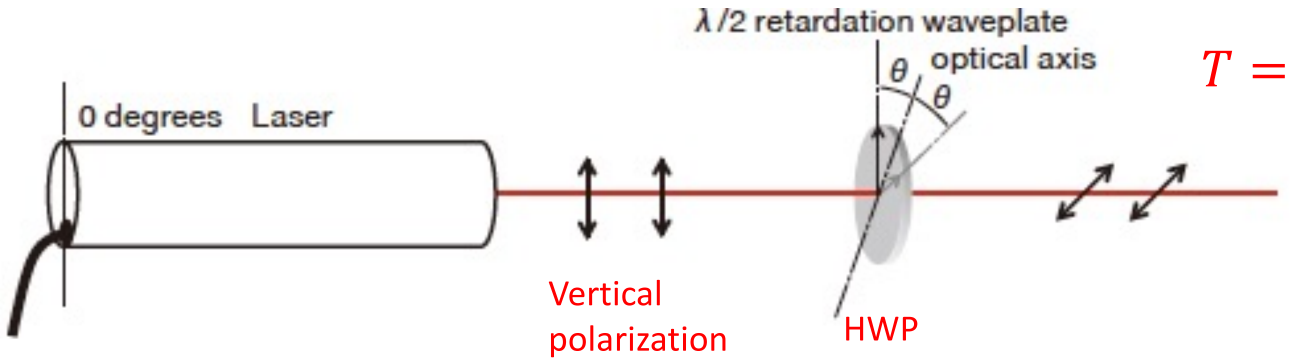
Operator of HWP (its axis is at angle θ with the vertical direction)

$$T = e^{i\frac{\pi}{2}}\begin{bmatrix}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{bmatrix}$$





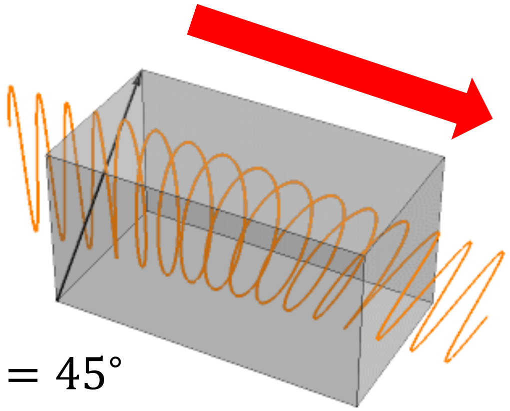
Your 1st Q gate realization using half-wave plate (HWP)



$$T = e^{i\frac{\pi}{2}}\begin{bmatrix}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{bmatrix}$$

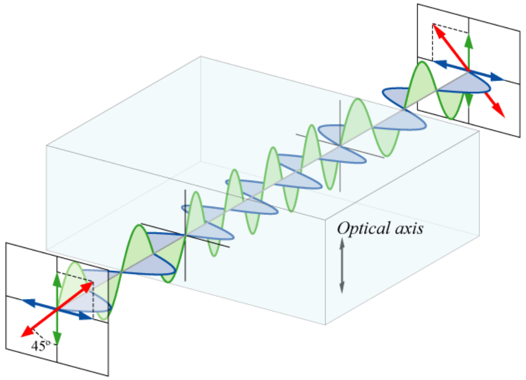
Example: Single photon with vertical polarization state and HWP with $\theta = 45\ deg.$:

$$|\psi_{in}\rangle = |V\rangle \equiv |1\rangle \equiv \begin{bmatrix}0\\1\end{bmatrix} \xrightarrow[\text{HWP action}]{\mathrm{T}(45^\circ)} |\psi_{out}\rangle = T|V\rangle = \begin{bmatrix}\cos 90^\circ & \sin 90^\circ \\ \sin 90^\circ & -\cos 90^\circ\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |H\rangle$$

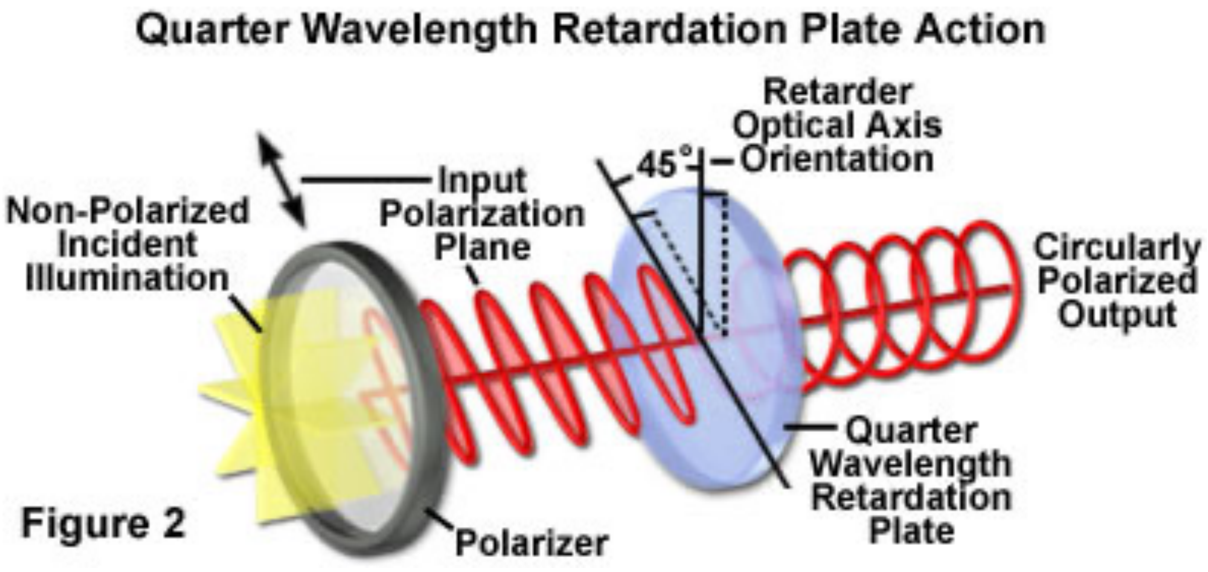


$\theta = 45^\circ$

- **Notice!!!** θ is under your full control in the Lab…
- For $\theta = 0\ deg.$, $\mathrm{T} = e^{i\frac{\pi}{2}}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} \equiv e^{i\frac{\pi}{2}}\mathrm{Z}$…The Pauli Z matrix (up to a global phase)
- For $\theta = 45\ deg.$, $\mathrm{T} = e^{i\frac{\pi}{2}}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \equiv e^{i\frac{\pi}{2}}\mathrm{X}$ …The NOT gate or Pauli X matrix
- For $\theta = 22.5\ deg.$, $\mathrm{T} = e^{i\frac{\pi}{2}}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix} \equiv e^{i\frac{\pi}{2}}\mathrm{H}$ …The Hadamard gate



Quarter-wave plate (QWP) acting on polarization state



$$T = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 - i\cos(2\theta) & -i\sin(2\theta) \\ -i\sin(2\theta) & 1 + i\cos(2\theta) \end{bmatrix}$$

Example: Single photon with diagonal polarization state and QWP with $\theta = 45\ deg.$:

$$|\psi_{in}\rangle = |H\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\xrightarrow{\mathrm{T}(45^\circ)}$

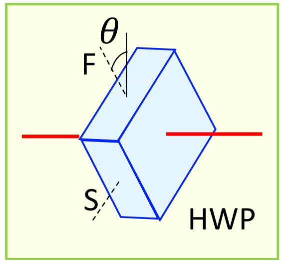
QWP action

$$|\psi_{out}\rangle = T|V\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

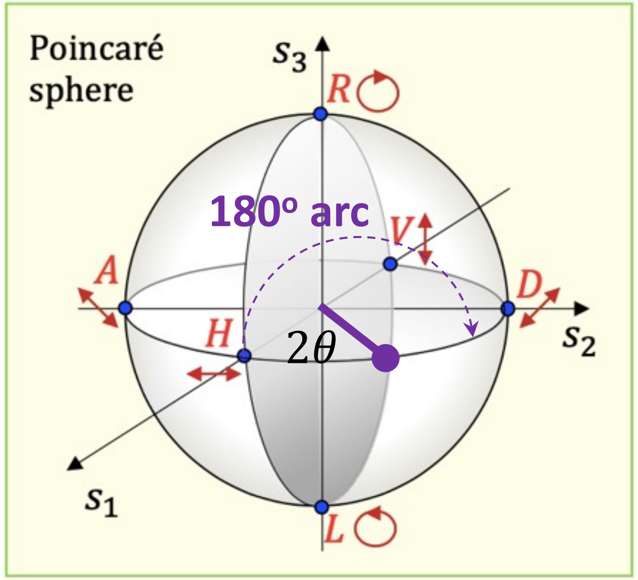
$$= \begin{bmatrix} -i \\ 1 \end{bmatrix} = -i\begin{bmatrix} 1 \\ i \end{bmatrix} = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle) = |R\rangle$$

Processing of polarization qubit

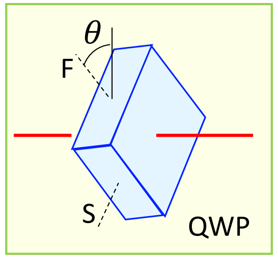
Half-wave plate



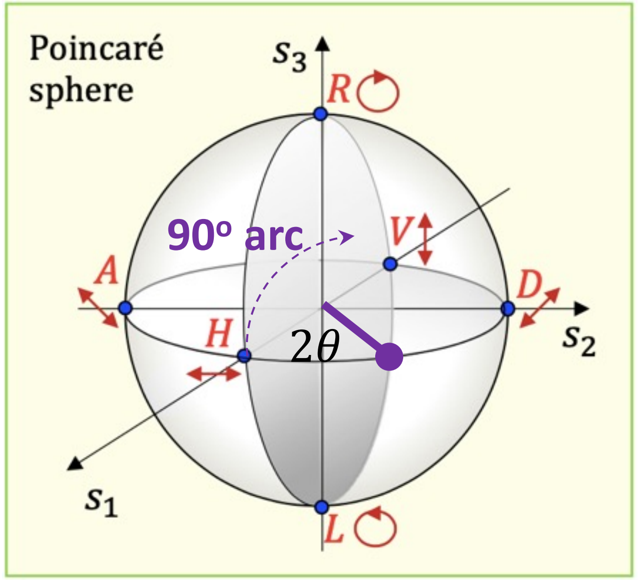
$$\text{HWP}(\theta) = \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$$



Quarter-wave plate



$$\text{QWP}(\theta) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 - i\cos(2\theta) & -i\sin(2\theta) \\ -i\sin(2\theta) & 1 + i\cos(2\theta) \end{bmatrix}$$



Operator	Matrix	Implementation
Pauli X	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	HWP at 45^o
Pauli Y	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	HWP at 0^o followed by HWP at 45^o
Pauli Z	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	HWP at 0^o
Hadamard H	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	HWP at 22.5^o
Balanced-symmetric SU(2) U_{BS}^{+}	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$	QWP at 45^o
Reflection $\text{R}_{\text{f}}(\theta)$	$\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$	HWP at $\theta/2$
Rotation-X $\text{R}_X(\theta)$	$\begin{bmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$	Wave plate $(\theta/2)$ at 45^o or QWP+$\text{R}_Y(\theta)$+QWP at 90^o
Rotation-Z $\text{R}_Z(\varphi) = e^{-i\varphi/2}\text{U}_{\text{p}}(\varphi)$	$\begin{bmatrix} e^{-i\varphi/2} & 0 \\ 0 & e^{i\varphi/2} \end{bmatrix}$	Wave plate φ at 0^o
Rotation-Y $\text{R}_Y(\theta) = \text{R}(\theta/2)$	$\begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$	Polarization rotator θ or HWP at $\theta/4$ followed by HWP at 0^o

$$\text{QWP}(\theta_3)\,.\,\text{HWP}(\theta_2)\,.\,\text{QWP}(\theta_1)$$

Any arbitrary $U(2)$ operation

“Quantum Photonics”, B. Saleh, just published in June 2025

Photonic Qubits can be realized in time

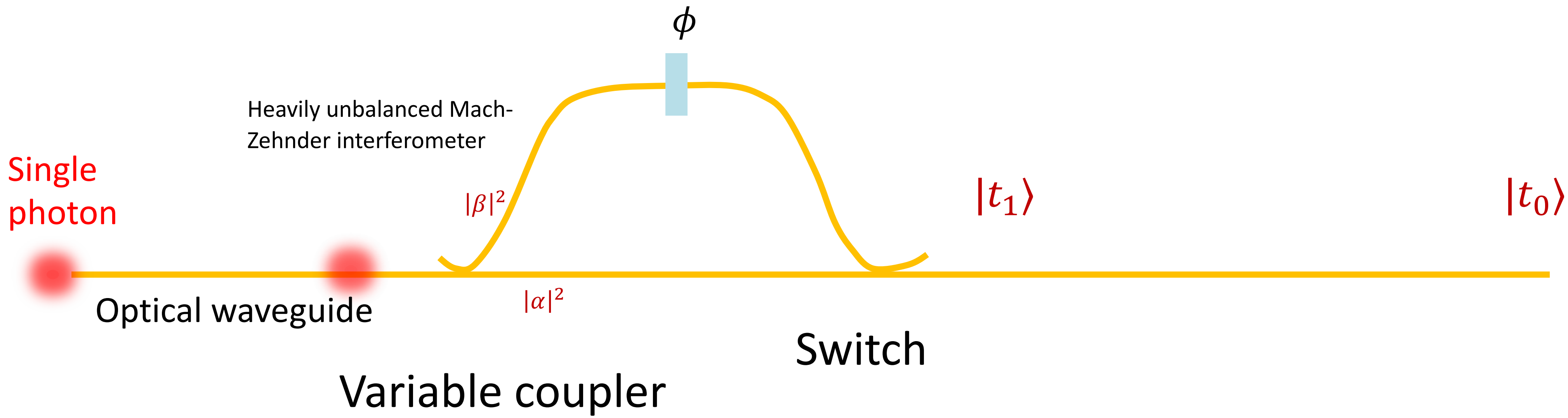
Temporal modes: Time is a continuous domain. Time bins (where portions of the photon wavefunction are distributed between well-separated time instants labelled, e.g., $|t_0\rangle$, $|t_1\rangle$ …, $|t_n\rangle$) represent discretized modes of a temporal wavefunction, and form a basis in the C^n space.

$$|\psi\rangle = \alpha|0\rangle + \beta e^{i\phi}|1\rangle$$

2 time modes

$$|\psi\rangle = \alpha|t_0\rangle + \beta e^{i\phi}|t_1\rangle$$

Time-bin qubit



* Red spot(s) represents photon wavefunction in space-time

Photonic Qubits can be realized in space

Spatial modes: space is a continuous variable, however, the spatial wavefunction can be represented as a superposition in discretized modes (orthogonal function) or paths (labelled, e.g., $|S_0\rangle$, $|S_1\rangle$, …$|S_{n-1}\rangle$) and form a basis in C^n space.

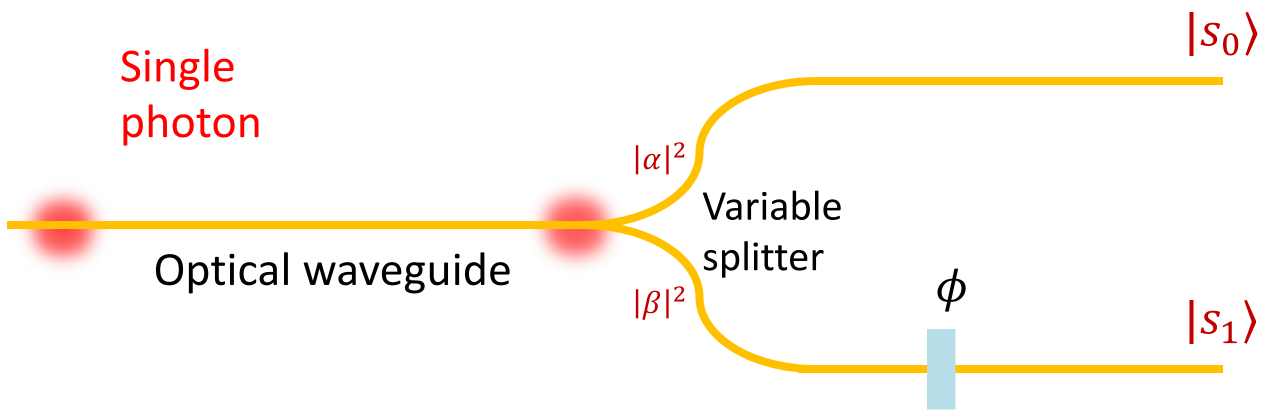
$$|\psi\rangle = \alpha|0\rangle + \beta e^{i\phi}\,|1\rangle$$

two spatial modes

Path mode, plane-wave modes … etc.

Spatial (or path) qubit

$$|\psi\rangle = \alpha|s_0\rangle + \beta e^{i\phi}|s_1\rangle$$

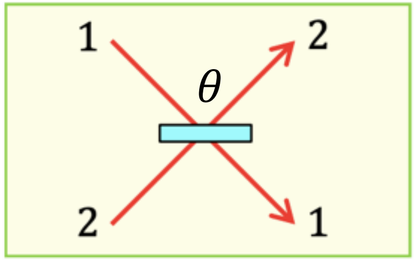


Dual-rail qubit

* Red spot(s) represents photon wavefunction in space-time

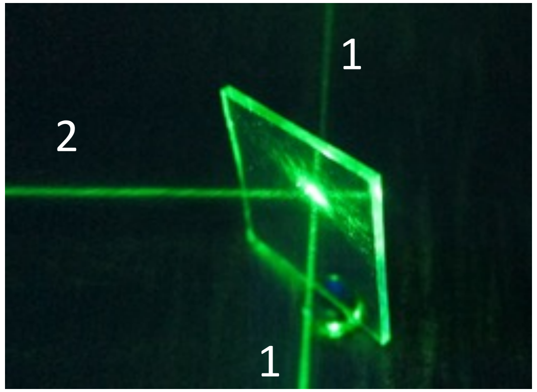
Qubit Processing in path domain

Symmetric Beamsplitter

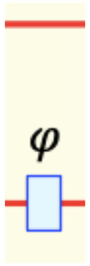


$$BS(\theta) = \begin{bmatrix} \cos\theta & -i\sin\theta \\ -i\sin\theta & \cos\theta \end{bmatrix}$$

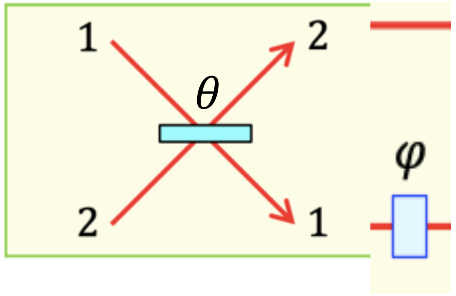
Reflectance of the beamsplitter : $\sin^2\theta$
Transmittance : $\cos^2\theta$



Phase shift



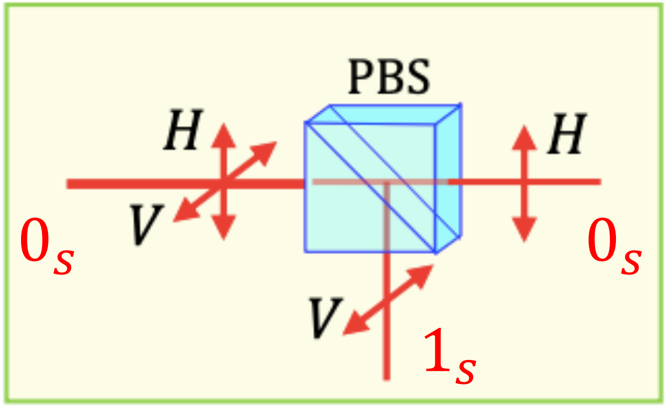
$$P(\phi) = \begin{bmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{bmatrix}$$



$$P(\phi)\,.\,BS(\theta) = \begin{bmatrix} e^{i\phi}\cos\theta & -i\sin\theta \\ -ie^{i\phi}\sin\theta & \cos\theta \end{bmatrix}$$

Any arbitrary $U(2)$ operation

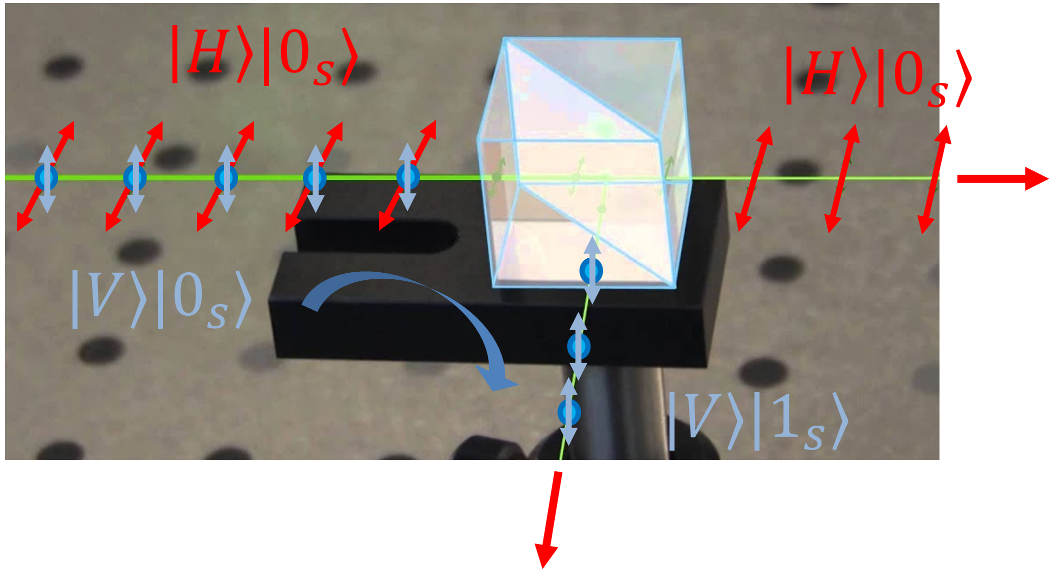
All what we have so far are single-qubit gates, what about two-qubit gates like CNOT



$$\mathrm{PBS}_{p\otimes s} \leftrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \equiv \mathrm{cNOT}$$

Polarizing Beam splitter (PBS)

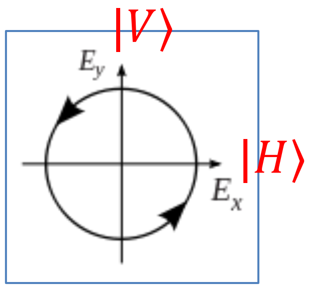
Single photons



Qubits can be realized in different degrees of freedom of light

Polarization qubit

$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$



Time-bin qubit

$|\psi\rangle = \alpha|t_0\rangle + \beta|t_1\rangle$

Path (dual-rail) qubit

$|\psi\rangle = \alpha|s_0\rangle + \beta|s_1\rangle$

Fock-state qubit

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

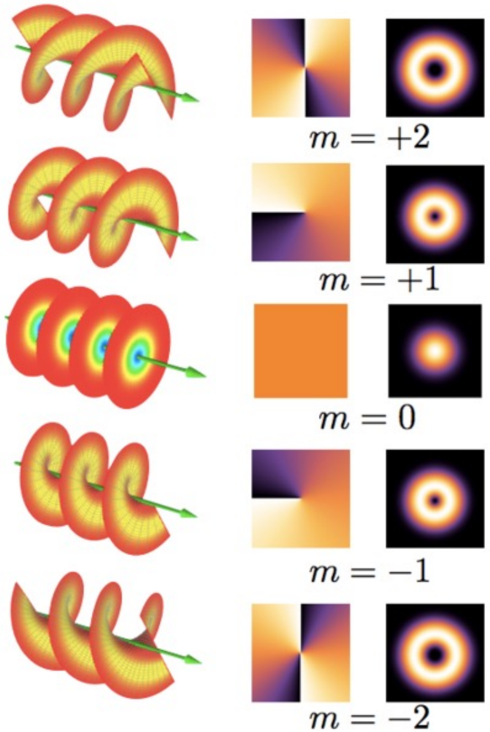
No Photons

One Photon

Orbital Angular Momentum (OAM) qudit

$|\psi\rangle = \alpha|\ell_{-2}\rangle + \beta|\ell_{-1}\rangle + \gamma|\ell_0\rangle + \delta|\ell_1\rangle + \kappa|\ell_2\rangle$

OAM modes



Two Qubits

Two classical bits have four possible states, 00, 01, 10, and 11, what about two qubits?

2 qubits: A,B

$$|\psi\rangle = \alpha_{00}|0_A\rangle|0_B\rangle + \alpha_{01}|0_A\rangle|1_B\rangle + \alpha_{10}|1_A\rangle|0_B\rangle + \alpha_{11}|1_A\rangle|1_B\rangle$$

- Coherent superposition in 4-D space
- $\alpha_{00}, \alpha_{01}, \alpha_{10}, \alpha_{11}$ are complex numbers (amplitudes).
- $|\psi\rangle$ is normalized, so $\sqrt{|\alpha_{00}|^2+|\alpha_{01}|^2+|\alpha_{10}|^2+|\alpha_{11}|^2} = 1$

The hidden 'information' in α_{ij} grows exponentially with the number of qubits

For 3 qubits, we have 8 coefficients.

For 500 qubit, we have 2^{500} coefficient… larger than the estimated number of atoms in the Universe*

This exponential increase in quantum information with no. of qubits, widely known to be underlying the quantum supremacy

* Quantum computation quantum information, 2010, by Nielsen & Chuang

Special two-qubit states: maximally entangled states

2 qubits: A,B

$$\alpha_{00}|0_A\rangle|0_B\rangle + \alpha_{01}|0_A\rangle|1_B\rangle + \alpha_{10}|1_A\rangle|0_B\rangle + \alpha_{11}|1_A\rangle|1_B\rangle$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

Widely used in many quantum applications:

- Quantum Teleportation
- Quantum encryption
- Quantum Computation
- Quantum superdense coding

....etc

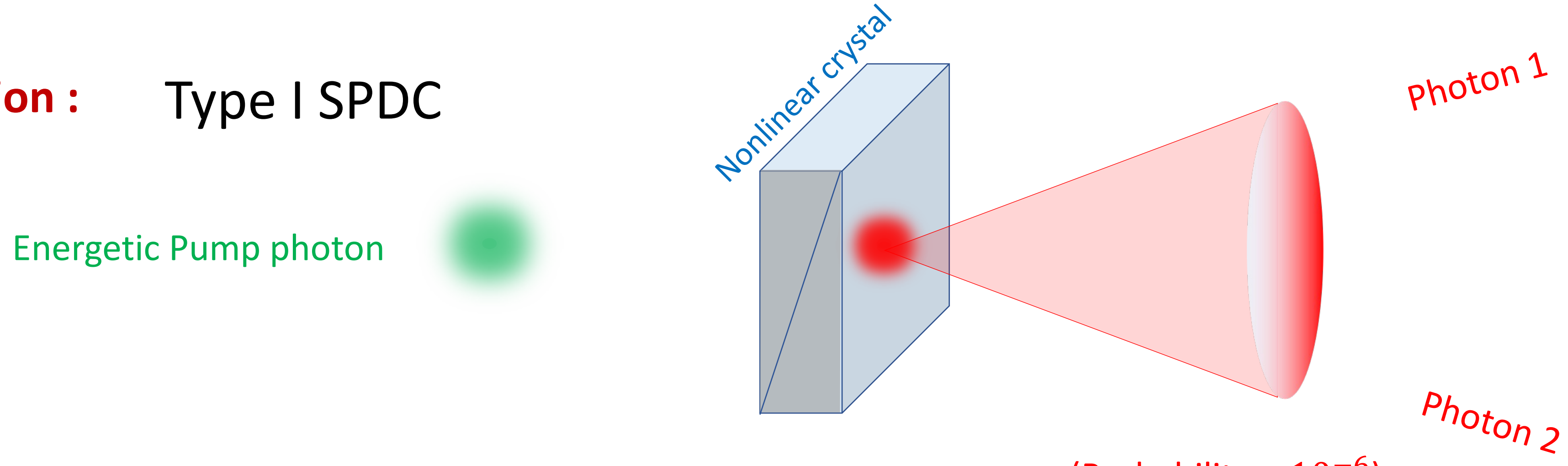
Bell states: are orthogonal in 4-D Hilbert space C^4

Generation of Entangled *photon* qubits

Spontaneous parametric downconversion (SPDC)

By illuminating a nonlinear crystal by an intense laser beam,
some photons split into two daughter photons (traditionally named **signal** and **idler** photons).

Nonlinear interaction : Type I SPDC



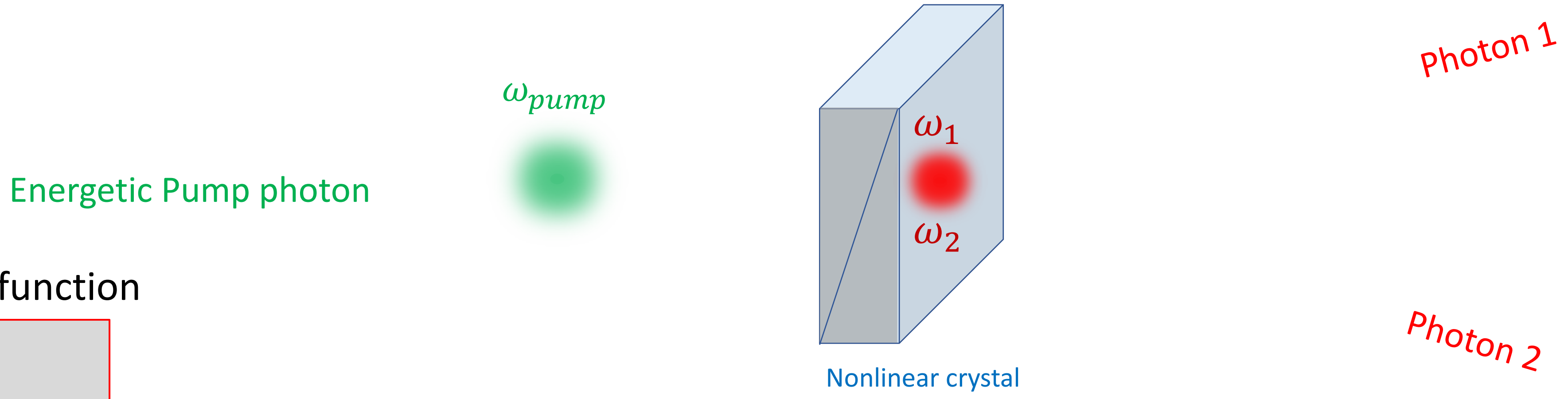
$V \rightarrow H_1H_2$ (Probability < 10^{-6})

- Conservation of energy : $\hbar\omega_{\text{pump}} = \hbar\omega_1 + \hbar\omega_2$
- Conservation of momentum : $\hbar\,\mathbf{k}_{\text{pump}} = \hbar\,\mathbf{k}_1 + \hbar\,\mathbf{k}_2$

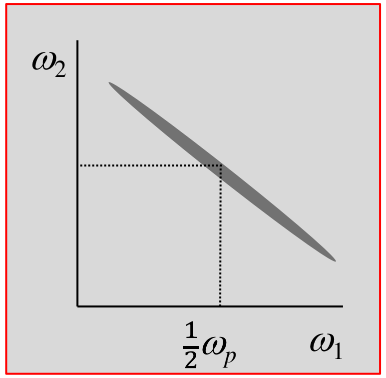
Generation of Entangled *photon* qubits

- **Frequency-entangled photons**

Conservation of energy: $\hbar\omega_{pump} = \hbar\omega_1 + \hbar\omega_2$ ($\hbar\omega$ is Photon energy, ω is angular frequency)



Spectral wavefunction



We are totally uncertain about the values of frequencies ω_1, ω_2.
Once one of them is measured, the other will immediately have a certain value

$$\omega_2 = \omega_{pump} - \omega_1$$

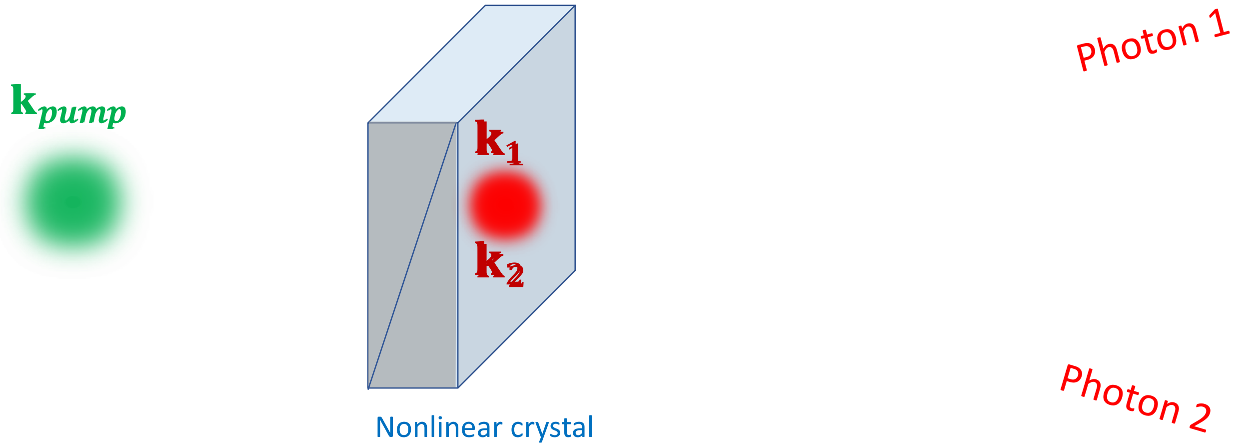
(Spectral state collapse)

Generation of Entangled *photon* qubits

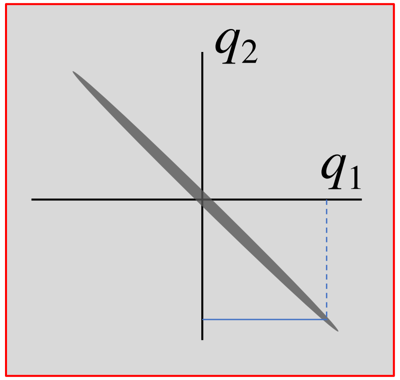
- **Momentum-entangled photons**

Conservation of momentum: $\hbar \mathbf{k}_{pump} = \hbar \mathbf{k}_1 + \hbar \mathbf{k}_2$ ($\hbar$ $\mathbf{k}$ is Photon momentum, $\mathbf{k}$ is the wavevector)

Energetic Pump photon



Transverse momentum wavefunction



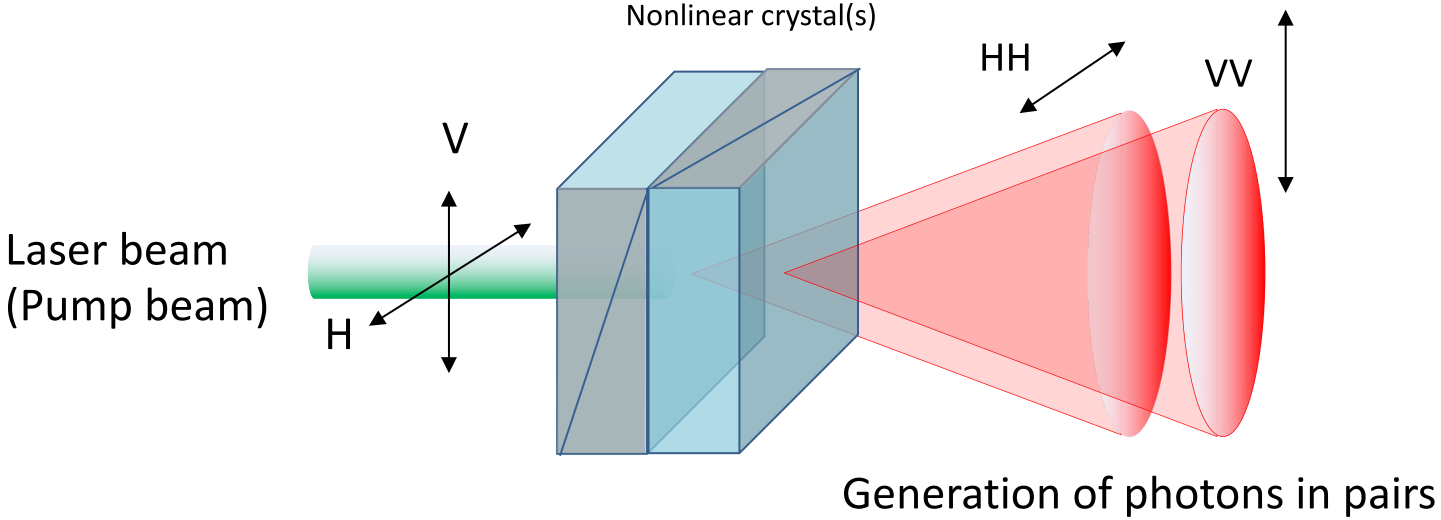
We are totally uncertain about the values of momenta $\mathbf{k}_1, \mathbf{k}_2$.
Once one of them is measured, the other will immediately have a certain value:

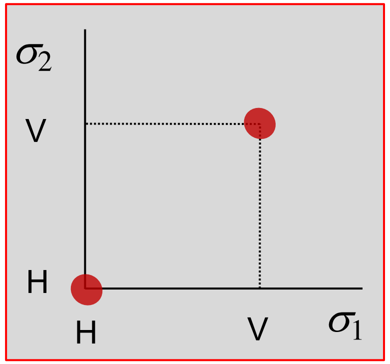
$$\mathbf{k}_2 = \mathbf{k}_{pump} - \mathbf{k}_1$$

(Momentum state collapse)

Generation of Entangled photons

- **Polarization-entangled photons**





σ : Polarization

Nonlinear interaction:

$$V \rightarrow H_1H_2 \quad + \quad H \rightarrow V_1V_2$$

$+$ Vertical polarized pump → horizontal polarized pair of photons

Horizontal polarized pump → Vertical polarized pair of photons

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}|H_1H_2\rangle + \frac{1}{\sqrt{2}}|V_1V_2\rangle$$

Polarization-entangled state

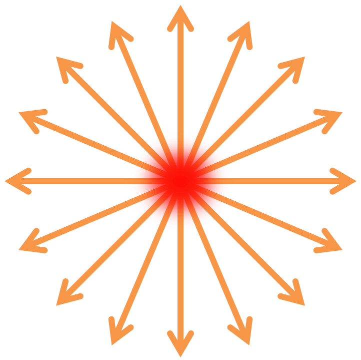
Two polarization-entangled Photons

1st photon (50% ↔ , 50% ↕)

$$|\phi^{+}\rangle = \frac{1}{\sqrt{2}}|H_1 H_2\rangle + \frac{1}{\sqrt{2}}|V_1 V_2\rangle$$

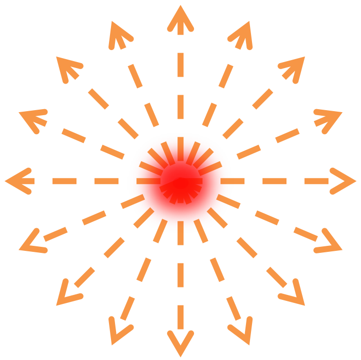
2nd photon (50% ↔ , 50% ↕)

- **Once one photon is measured,**
  - The state **collapses**: the value of the other qubit becomes certainly known.
  - Q Entanglement violates **locality** : There is an immediate (nonlocal) action at a distance
  - Q Entanglement does not violate **causality** : This can not transfer information



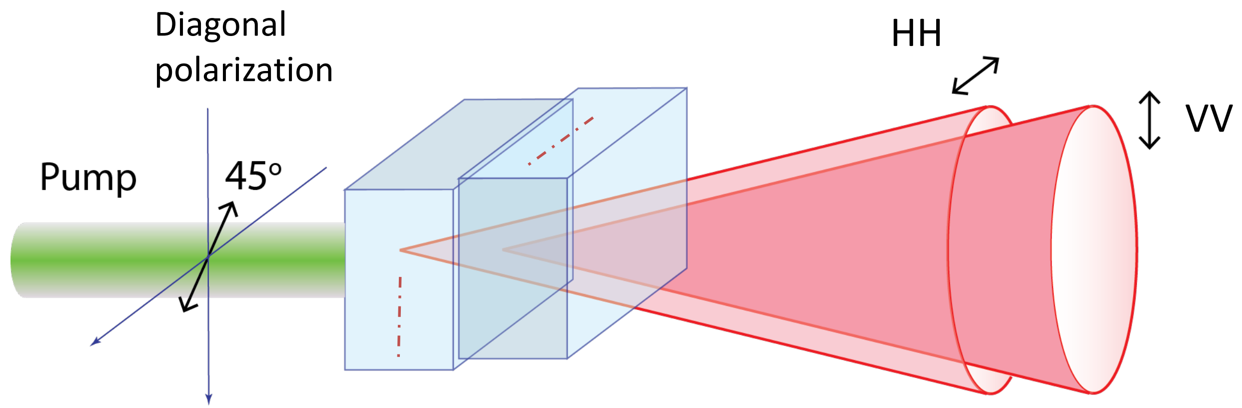
Polarization measurements 1st photon

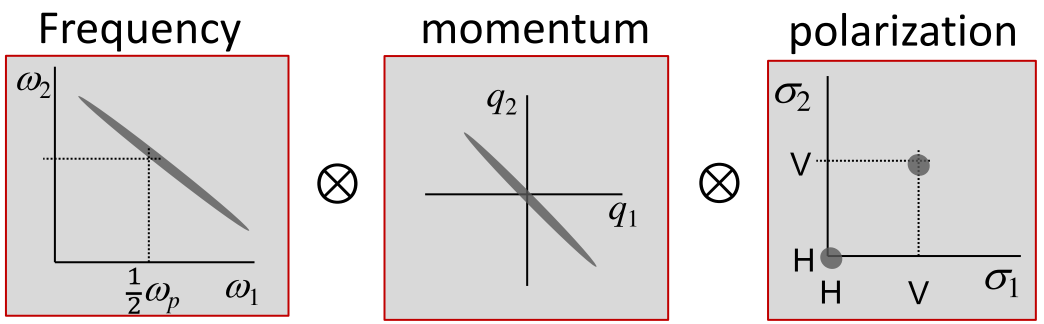
Photons are so far from each other



2nd photon

Hyperentangled Photon Sources: Cascaded Structure





Hyperentanglement :

Entanglement in every degree of freedom of light

- **Frequency entanglement**

Time Entanglement also exists:

Creation time is not known within the coherence time of the pump, until position is measured for one of the paired photons.

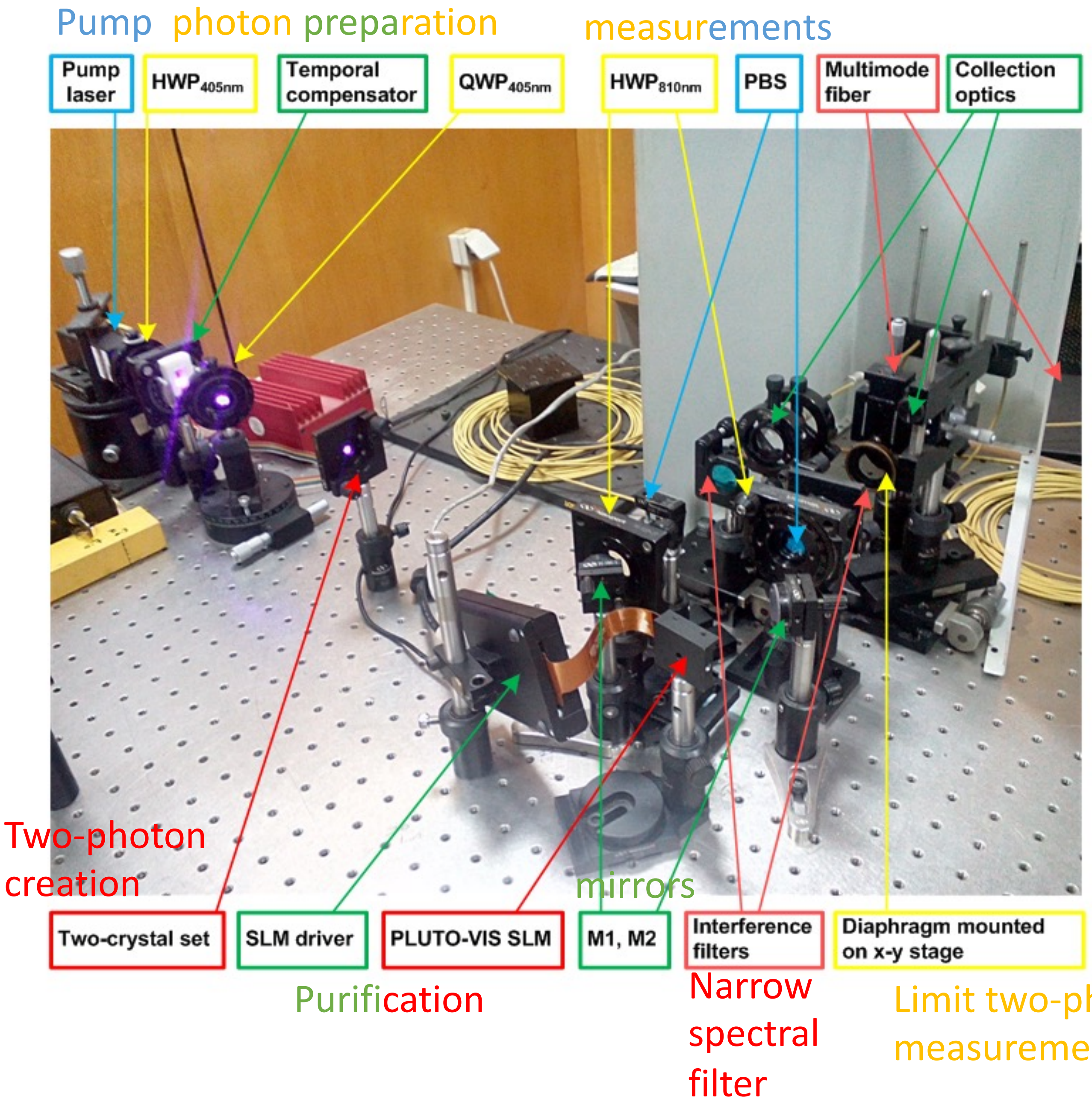
- **Momentum entanglement**

Position entanglement also exists :

Emission position is not known within the coherence width of the pump photon, until position is detected for one of the paired photons

- **Polarization entanglement**

Entangled photons generation, purification, measurements



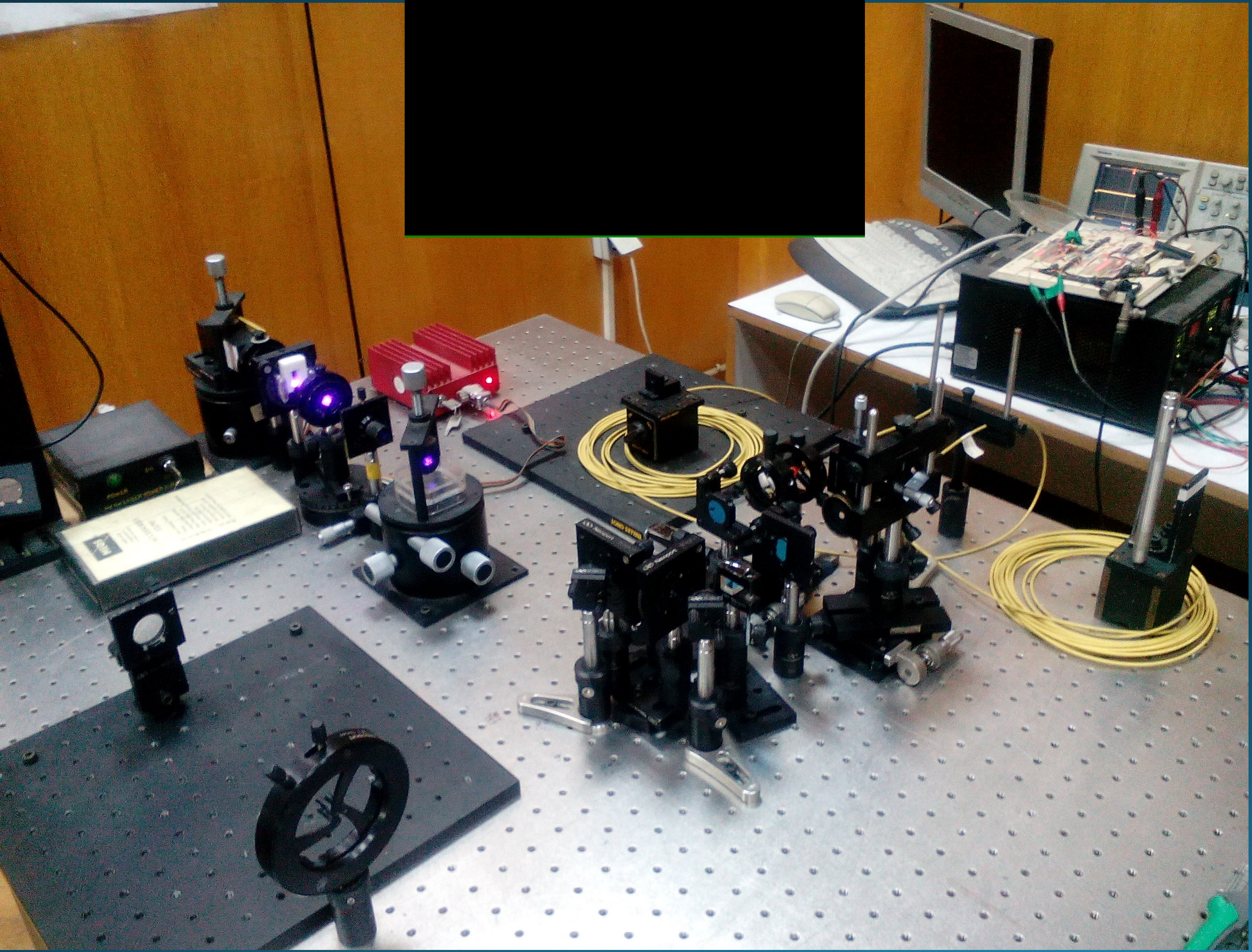
- Bell inequality was violated by a record :

$$2.618 \pm 0.06$$

- Photon pairs are entangled in polarization, momentum & frequency

- Polarization-entangled state :

$$\frac{1}{\sqrt{2}}\left(|H_1H_2\rangle + e^{i\phi}\,|V_1V_2\rangle\right)$$



Experimental setup

- Two-crystal: 0.5 mm BBO
- Pump laser: 405 nm, 30 mw
- Detection filter: 10-nm centered at 810 nm
- Multimode fiber detection
- 2.6 deg. collection angle

Result

Coincidence counts: $4{,}100\ s^{-1}$

Single counts: $139{,}200\ s^{-1}$

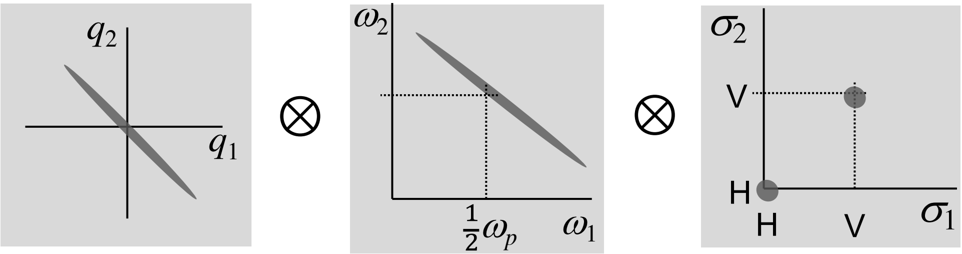
Background counts : $95{,}300\ s^{-1}$

Collection efficiency:

$$\frac{4{,}100\ s^{-1}}{139{,}200\ s^{-1} - 95{,}300\ s^{-1}} = 9.3\%$$

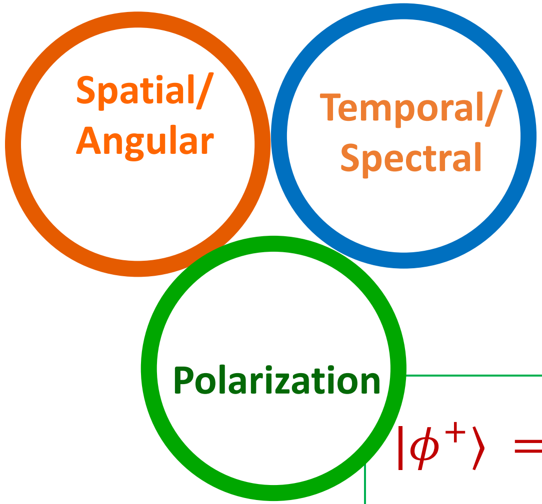
Hyperentanglement : Entanglement in multiple degrees of freedom (DoFs)

Momentum | Frequency | Polarization



Two-photon wavefunction in every DOF

Hyperentanglement, Ideally



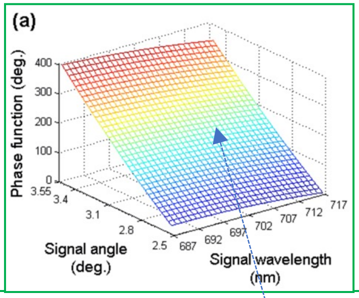
$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|H_1H_2\rangle + |V_1V_2\rangle)$$

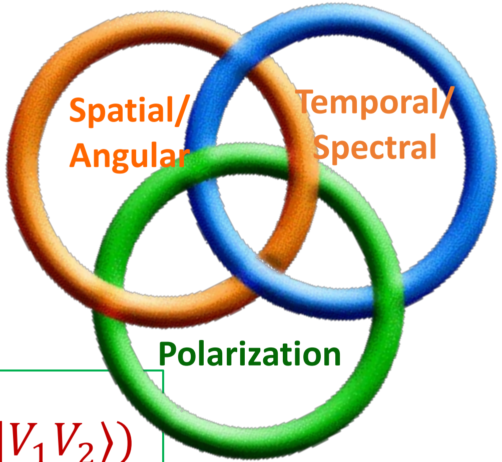
Independent degrees of freedom (DoFs)

→ High purity entangled state
→ No Errors

Effects such as birefringence, dispersion in nonlinear crystal couple these DoFs

Actual source





$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|H_1H_2\rangle + e^{i\phi(\omega,q)}|V_1V_2\rangle)$$

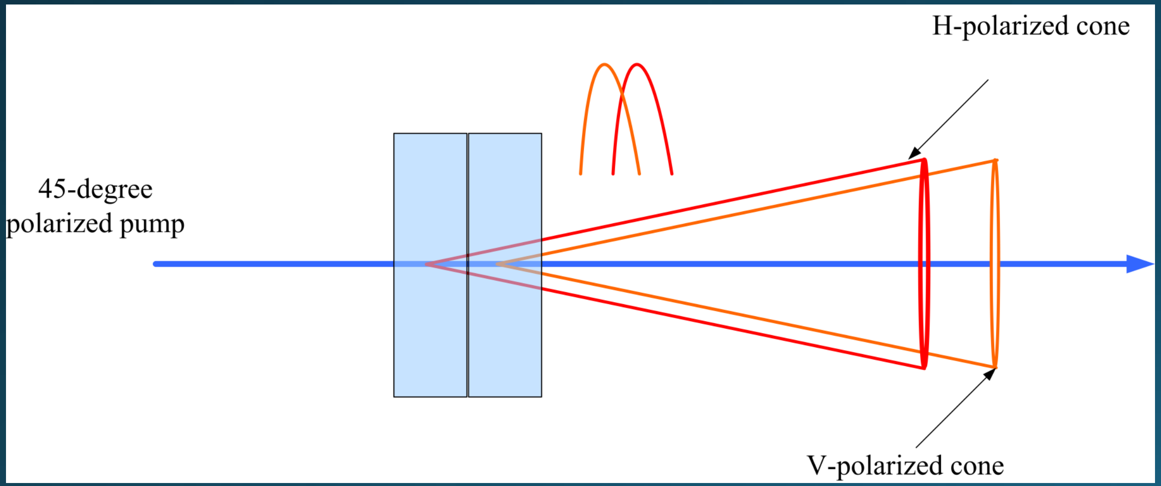
coupling inherent between photon's degrees of freedom (DoFs)

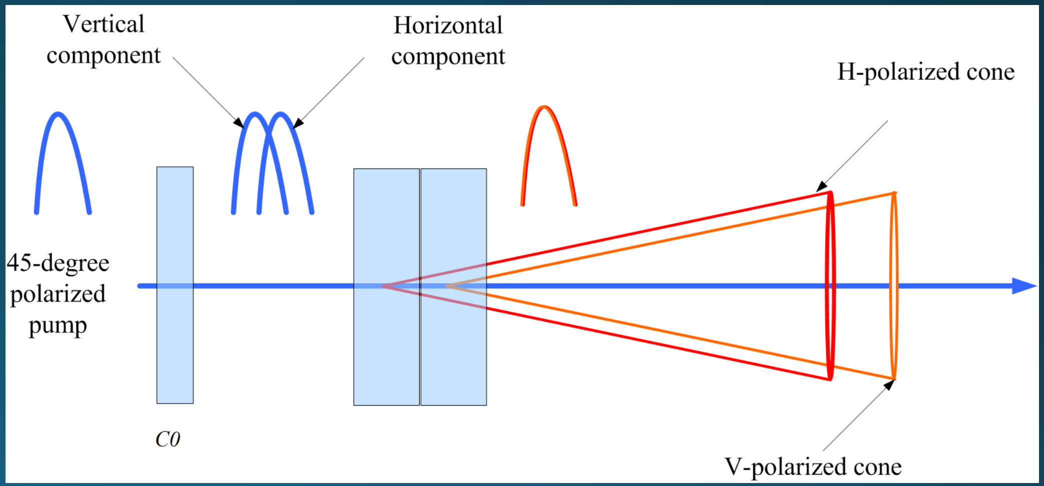
→ Drop in Entanglement purity
→ Errors in quantum Applications

Tunable Spatial-Spectral Phase Compensation of Type-I (ooe) Hyperentangled Photons

Output state decoherence:

2- Temporal decoherence: limits the overlap of the photon pairs emitted by 1st and 2nd crystals. Important!!! when low coherence-time pump (such as diode laser and femtosecond laser) is used.



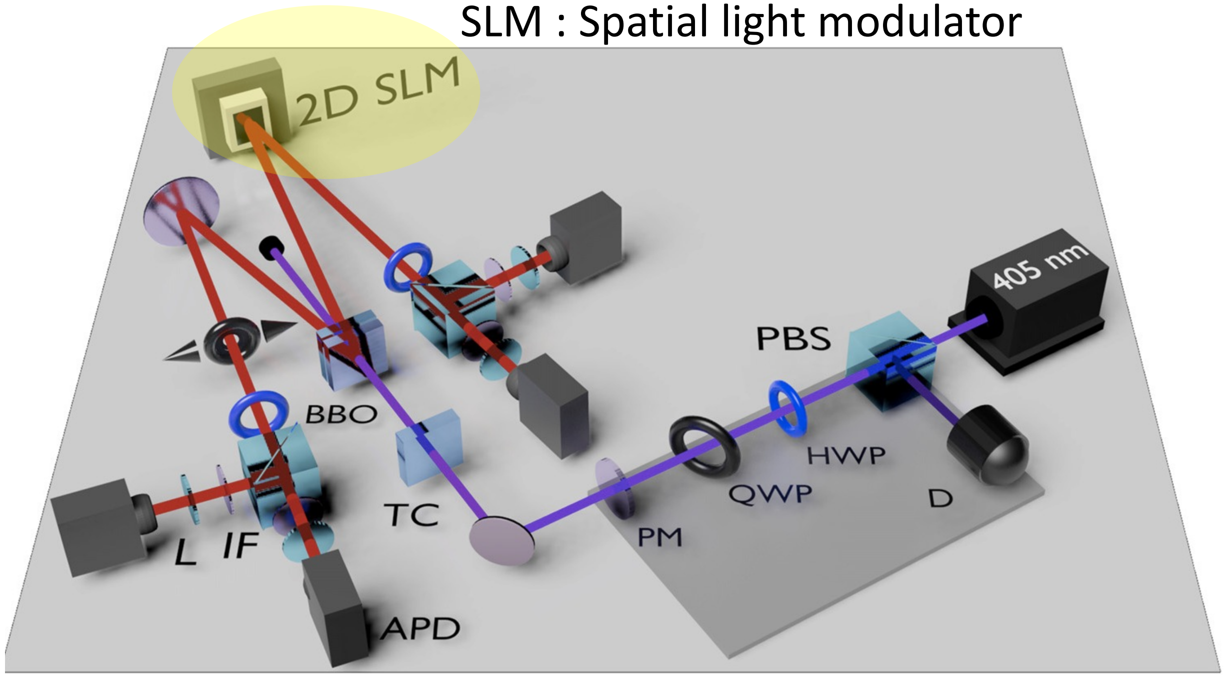


Two-crystal source with (to the right) and without (to the left) temporal compensation

“Tunable spatial–spectral phase compensation of type-I (ooe) hyperentangled photons”. JOSA B, 32(3), 445-450 (2015).

Purification of entangled photons **over wide angles of emission – using SLM**

SLM : Spatial light modulator



Experimentally verified expression for the *Phase* function over the SPDC cone

$$\phi_{DC}(\omega_1,\omega_2;\mathbf{q}_1,-\mathbf{q}_1) \approx \Bigg\{ \frac{(n_p^o - n_1^{e\perp})\omega_1 + (n_p^o - n_2^{e\perp})\omega_2}{c} - q_{1,x}(\tan\rho_1^{\perp} - \tan\rho_2^{\perp}) + \frac{c q_{1,x}^2}{2}\left[\frac{(n_1^{e\perp})^3}{\omega_1 (n_1^e)^2 (n_1^o)^2} + \frac{(n_2^{e\perp})^3}{\omega_2 (n_2^e)^2 (n_2^o)^2}\right] + \frac{c q_{1,y}^2}{2}\left[\frac{n_1^{e\perp}}{\omega_1 (n_1^e)^2} + \frac{n_2^{e\perp}}{\omega_2 (n_2^e)^2}\right]\Bigg\} L - \phi_p,$$

Variable in frequency and spatial domains

ϕ_{DC}

SLM additional phase ϕ_{SLM}

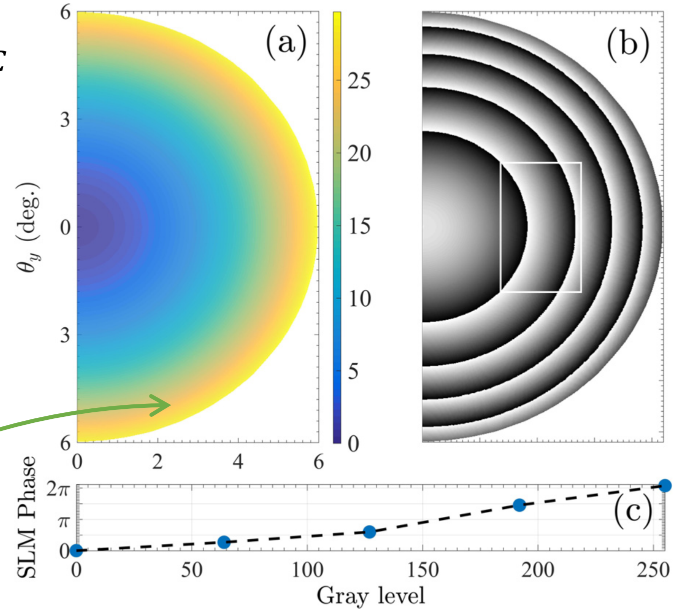


FIG. 3. (a) The two-dimensional quadratic relative-phase map (in radians) as predicted by Eq. (4) all over the SPDC cone. (b) Purifying SLM pattern to produce the states $|\phi^{\pm}\rangle$ with high fidelity over wide emission angles. The pattern constitutes the inverted *modulo-2π* grayscale of the map in (a). The white rectangle circumscribes the 2D SLM's active area on which the compensation image is loaded. This area covers azimuthal-angle range ~57° (of the 180° representing half SPDC cone); therefore, about one third of the noncollinear SPDC emission cone could be manipulated. (c) Experimentally measured phase retardation introduced by the 8-bit SLM at 810 nm plotted vs the gray display level (black:0 and white:255). The phase measurement error (±0.005π) is smaller than the readout markers.

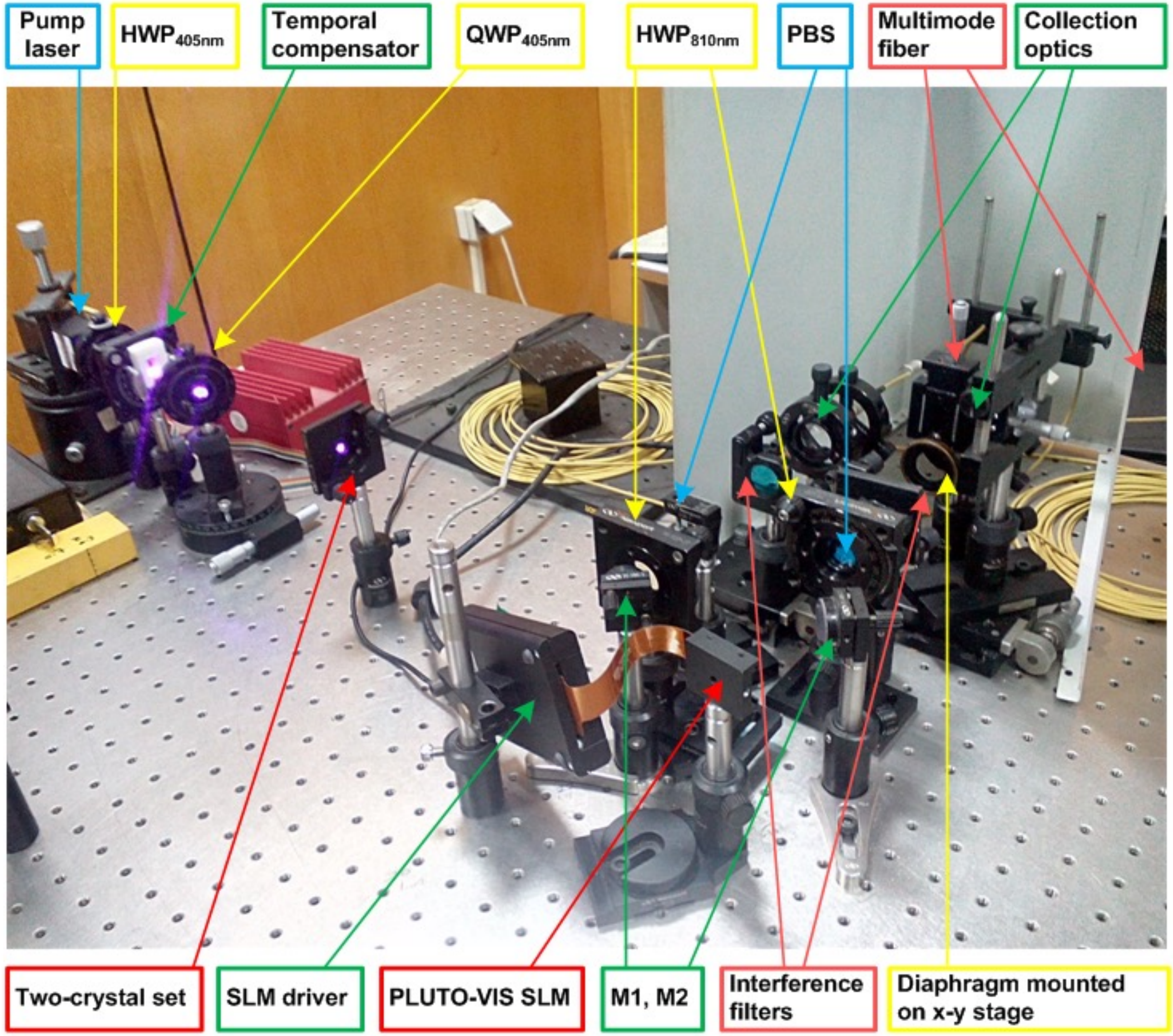
fixed in frequency and spatial domains

$$\{|H\rangle_1|H\rangle_2 + e^{i(\phi_{DC}+\phi_{SLM})}|V\rangle_1|V\rangle_2\}/\sqrt{2}$$

Optical Engineering 56(2), 026114-026114 (2017)

Purification of entangled photons **over wide angles of emission – using SLM**

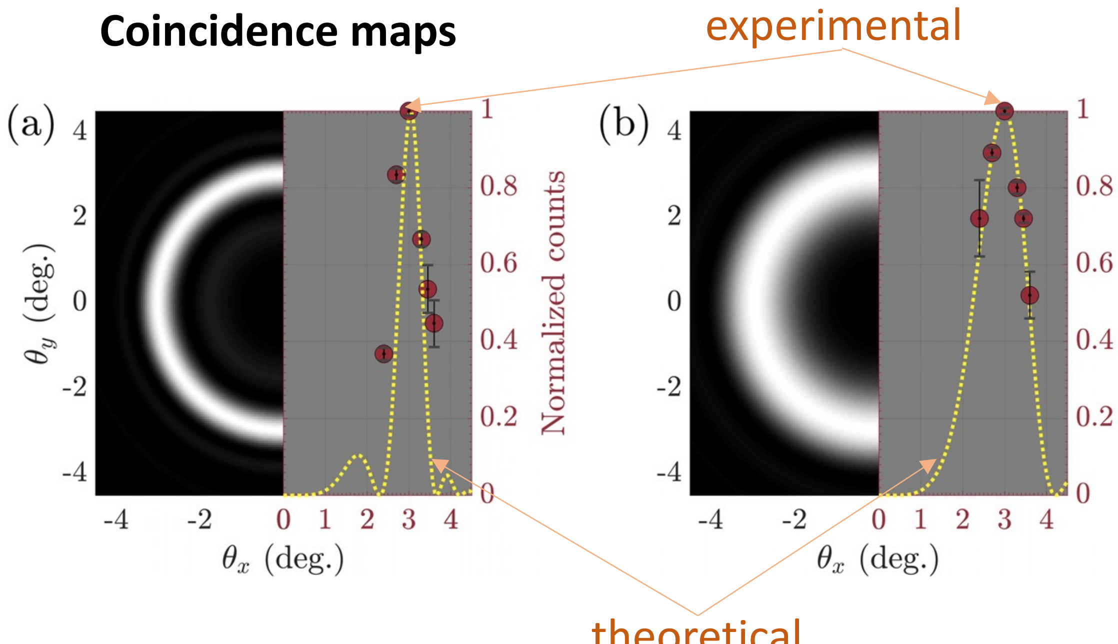
Experiment



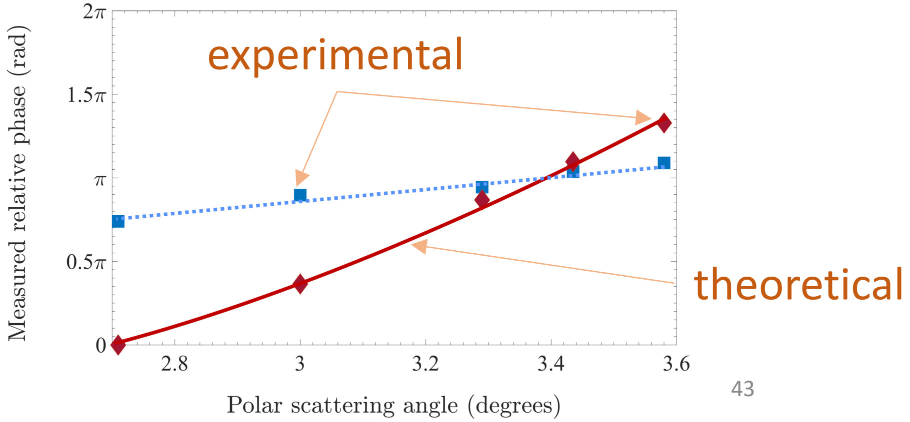
Applied Physics Letters 117, 244003 (2020)

Main experimental measurements

Coincidence maps



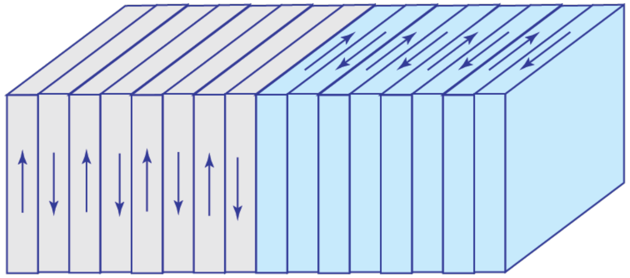
Relative phase



Novel Hyperentangled Photon Sources: **Superlattice Structure (SL)**

Is there a design of nonlinear structure that creates **highly pure** hyperentangled photons state

with no additional devices?



Periodically poled
Cascaded crystals
Orthogonal optic axes

US Patent application 62/489,044, 2017
Sci. Reports 7(1), 4169, 2017.
CLEO (San Jose, USA, 2017).

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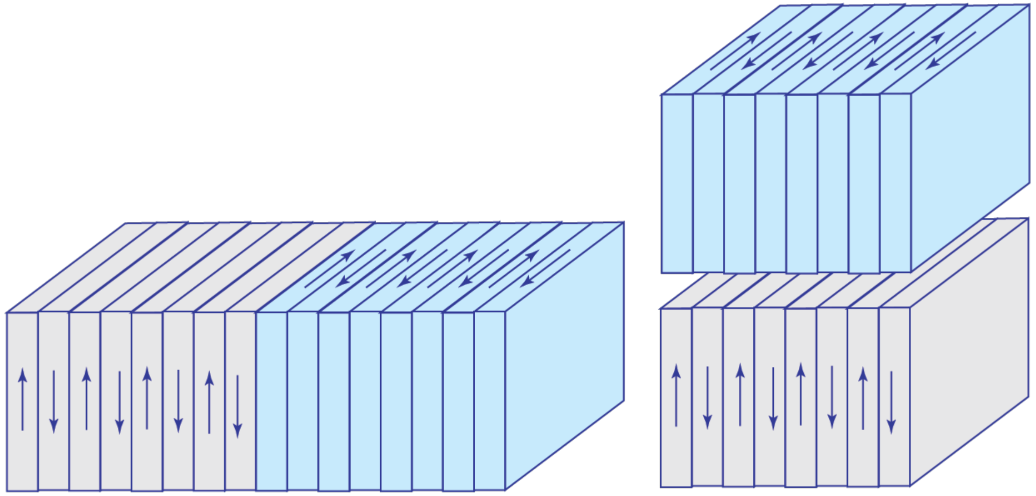

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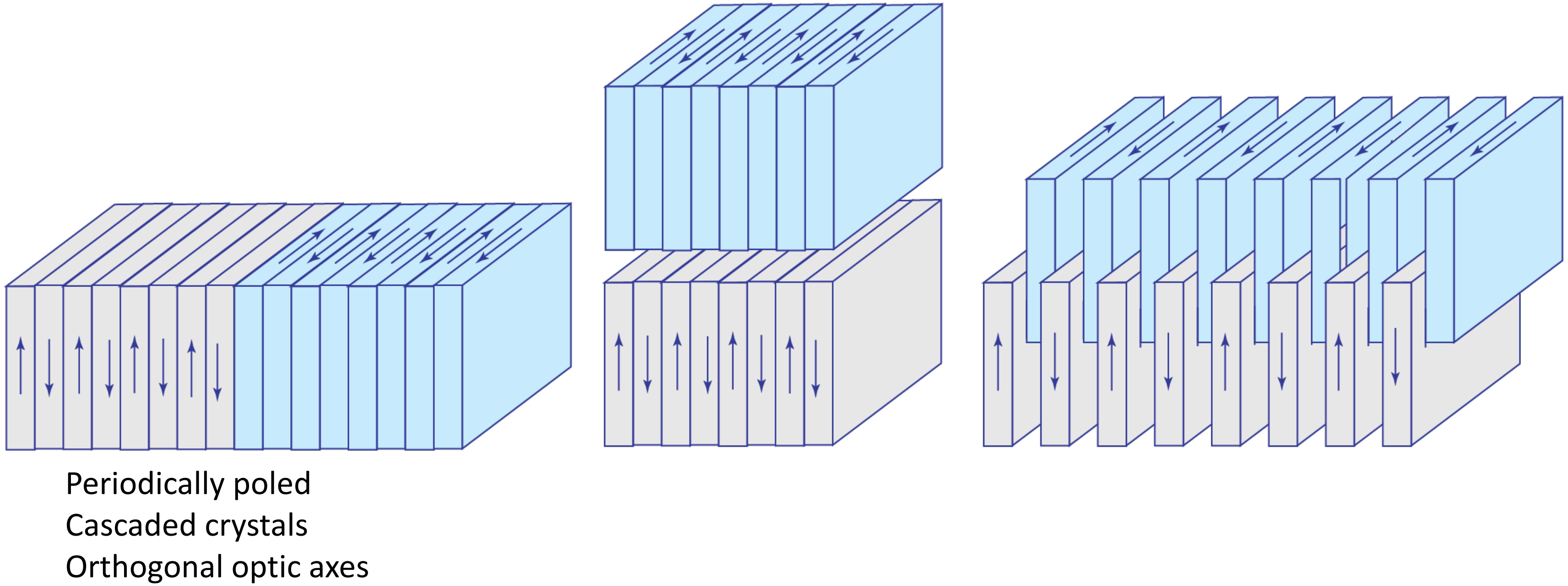

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Novel Hyperentangled Photon Sources: **Superlattice Structure (SL)**

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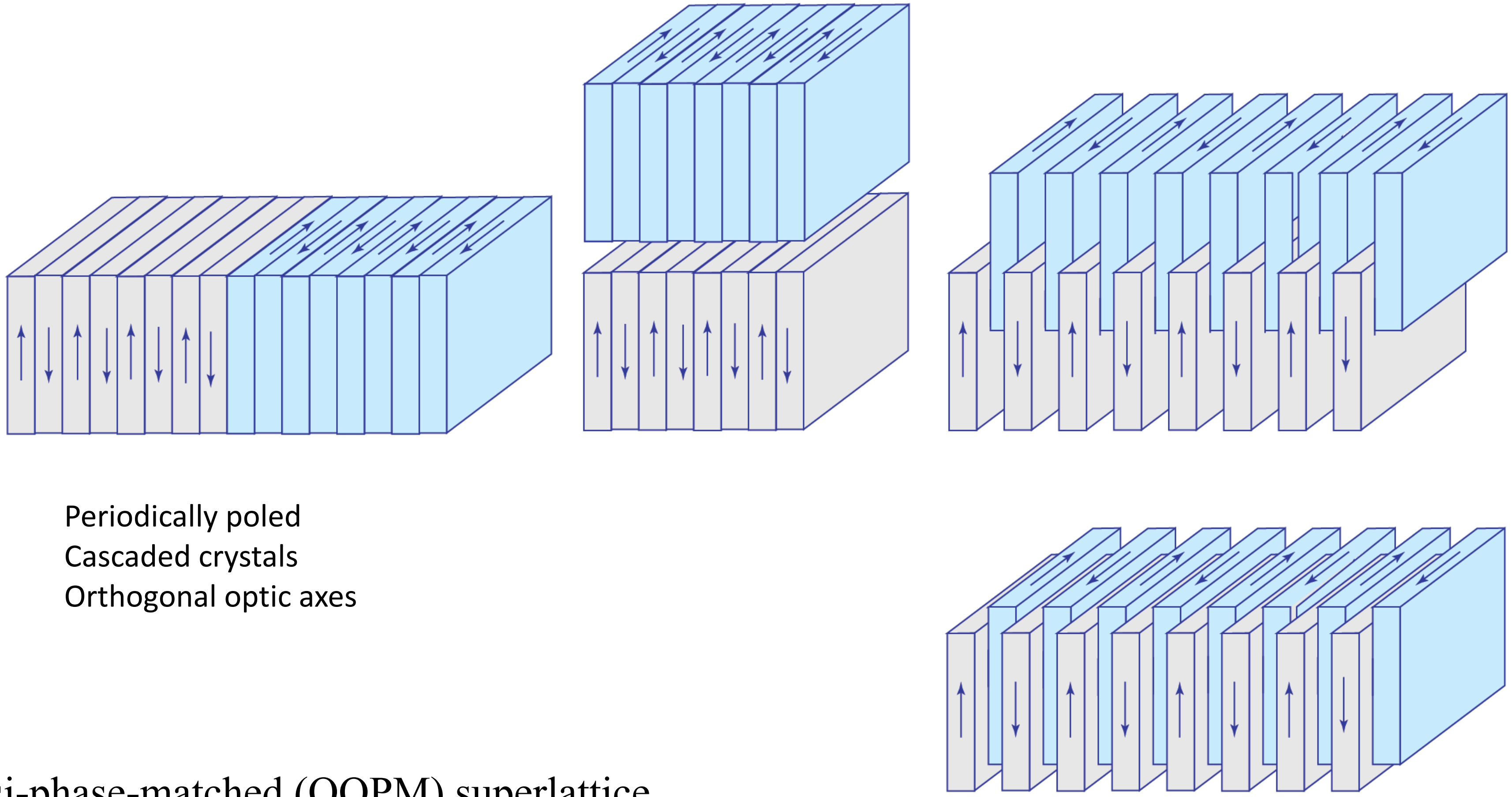

Alternating orthogonal optic axes interleaved with orthogonal periodic polling

We answered this question:

Yes, using the novel Orthogonal quasi-phase-matched (OQPM) superlattice

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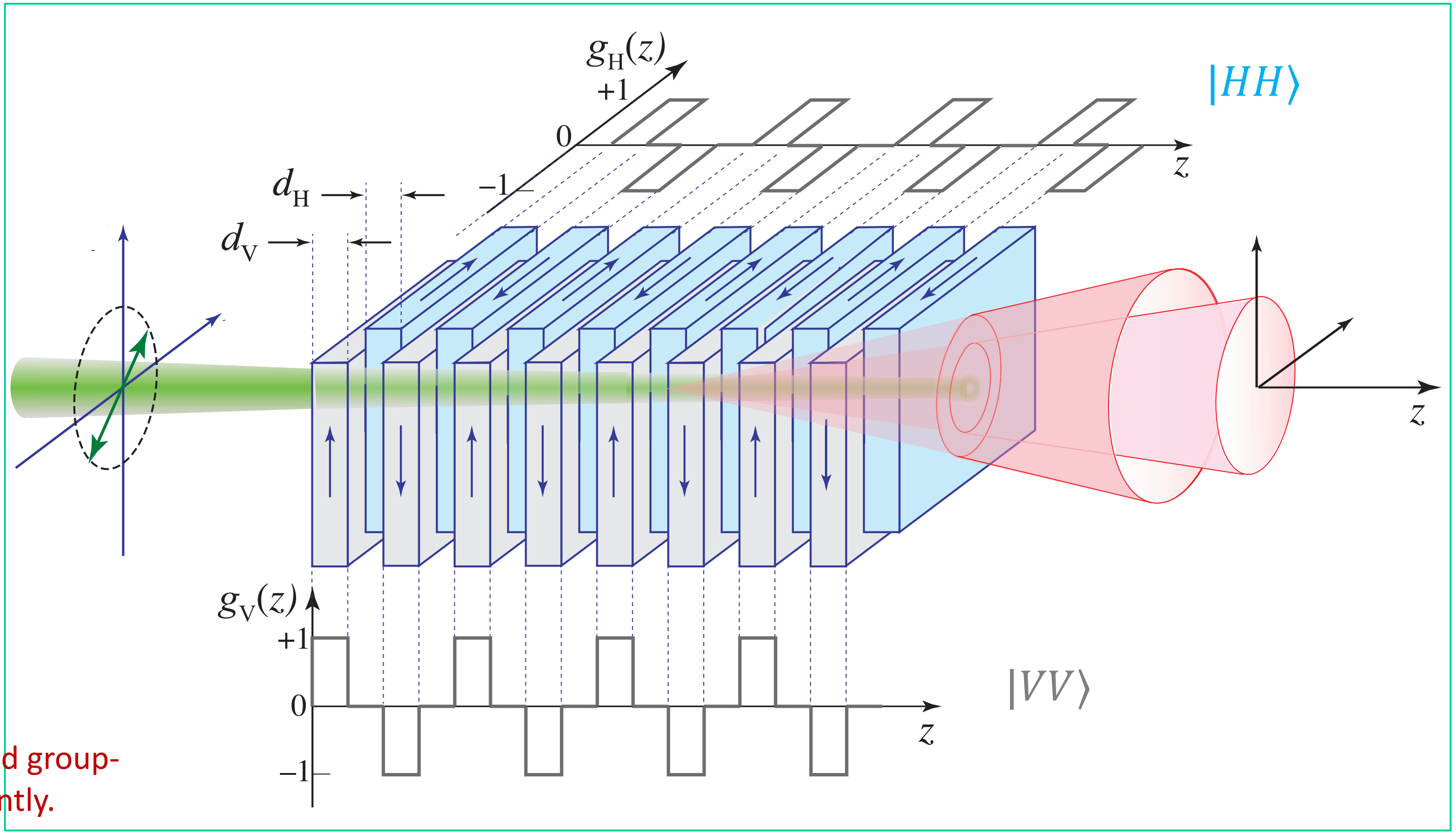
US Patent application 62/489,044, 2017
Sci. Reports 7(1), 4169, 2017.
CLEO (San Jose, USA, 2017).

Novel Hyperentangled Photon Sources: **Superlattice Structure (SL)**



SL structure corrects phase and group-velocity mismatches concurrently.

Output hyperentangled state

$$|\psi\rangle = \iint d\omega_1 d\omega_2 d\mathbf{q}_1 d\mathbf{q}_2 \left[\Phi_H(\omega_1,\omega_2;\mathbf{q}_1,\mathbf{q}_2)|H;\omega_1;\mathbf{q}_1\rangle|H;\omega_2;\mathbf{q}_2\rangle + \Phi_V(\omega_1,\omega_2;\mathbf{q}_1,\mathbf{q}_2)|V;\omega_1;\mathbf{q}_1\rangle|V;\omega_2;\mathbf{q}_2\rangle\right]$$

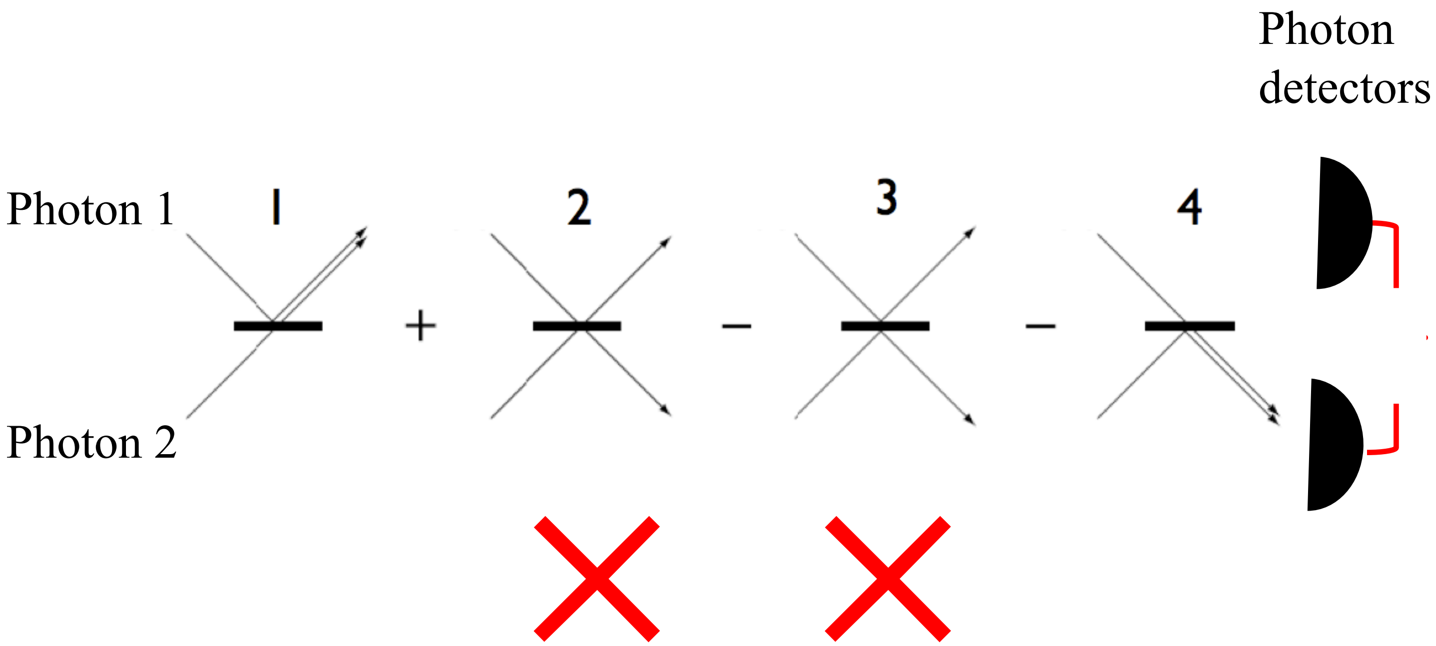
$$\Phi_{H,V}(\omega_1,\omega_2;\mathbf{q}_1,\mathbf{q}_2) \propto A_p(\omega_1+\omega_2;\mathbf{q}_1+\mathbf{q}_2)\ \mathrm{sinc}\left(\tfrac{1}{2}\Delta\kappa^e_{H,V} d_{H,V}\right) \frac{\sin\left(\frac{M}{4}\Delta\varphi_{H,V}\right)}{\sin\left(\frac{1}{2}\Delta\varphi_{H,V}\right)}$$

Pump — Two layers Cell — Structure

Phase-matching function

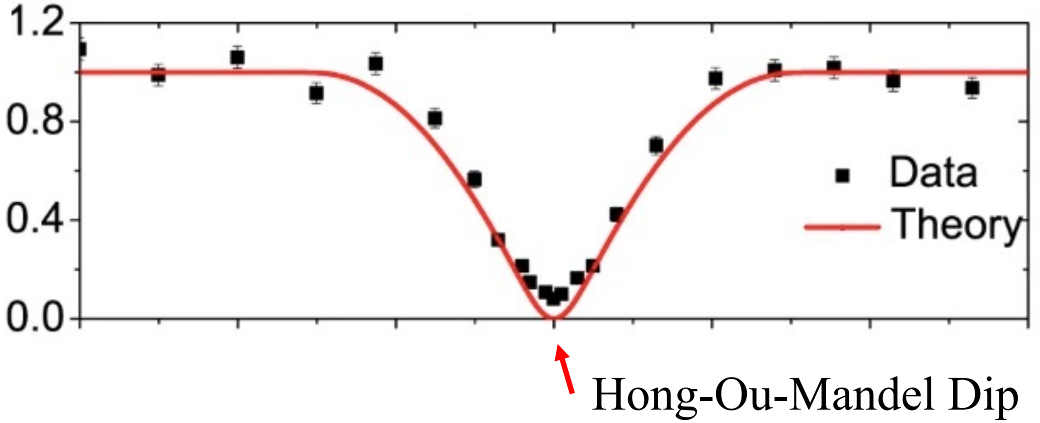
US Patent application 62/489,044, 2017
Sci. Reports 7(1), 4169, 2017.
CLEO (San Jose, USA, 2017).

Two-photon interference: a measure of temporal indistinguishability

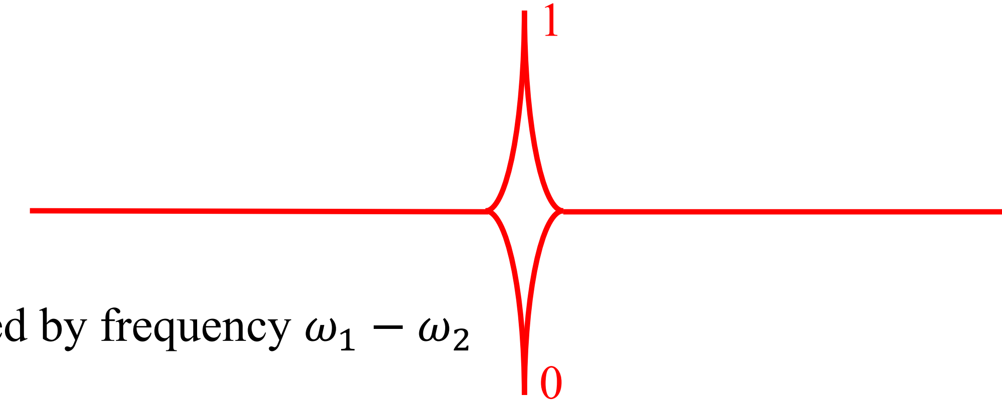


Two possibilities cancel each other for two identical photons

(a) Degenerate photons $\omega_1 = \omega_2$



(b) Non-degenerate photons $\omega_1 \neq \omega_2$



Dip modulated by frequency $\omega_1 - \omega_2$

Quantum information units in higher-dimensional spaces

Qubits

The physical system is associated with **2-dimensional (2D)** complex vector space

This means it includes 2 mutually orthogonal states (basis states)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$|0\rangle, |1\rangle$ are orthogonal
$\langle 0|1\rangle = 0$

Qutrits

3-dimensional complex vector space ($\mathbb{C}^3$)

This means it includes 4 mutually orthogonal states (basis states)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$$

$|0\rangle, |1\rangle, |2\rangle$
are orthogonal

Ququarts

4-dimensional complex vector space ($\mathbb{C}^4$)

This means it includes 4 mutually orthogonal states (basis states)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle + \delta|3\rangle$$

$|0\rangle, |1\rangle, |2\rangle, |3\rangle$
are orthogonal

Qudits

d-dimensional complex vector space